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An Empirical Analysis of Nikkei 225 Options Using Realized GARCH Models

Asuka Takeuchi-Nogimori

This paper analyses whether realized generalized autoregressive conditional heteroscedasticity (GARCH) models are useful for pricing Nikkei 225 options. This model enables us to estimate simultaneously the dynamics of stock returns using both realized volatility (RV) and daily return data. The analysis also examines whether realized GARCH models using realized kernels (RK) and realized ranges (RR) improve the option-pricing performance. Comparing the empirical results, for call options, EGARCH models perform better; however, for put options, realized GARCH models with RK without nontrading hour returns perform better than those with RV or RR.
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1. Introduction

One of the most important variables in option pricing is the volatility of the underlying asset, defined as the standard deviation of the returns of financial assets. However, while the well-recognized Blackand Scholes (1973) option-pricing model assumes that financial asset volatility is constant, it is well known that volatility changes over time. Giving consideration to the dynamic volatility process in the option-pricing model is necessary to evaluate options more accurately. Therefore, the purpose of this paper is to evaluate option prices using high-frequency data.

Many alternative time-series volatility models are now available to describe the dynamics of volatility. One traditional group of models is autoregressive conditional heteroscedasticity (ARCH)-type models using daily return data, including GARCH (generalized ARCH, Bollerslev, 1986), GJR (Glosten et al. 1993), EGARCH (exponential GARCH, Nelson, 1991), APGARCH (asymmetric power GARCH, Ding et al. 1993), and FIEGARCH (fractionally integrated EGARCH, Bollerslev and Mikkelsen, 1996) models. More recently, realized volatility (RV) models using high-frequency data have attracted the attention of financial econometricians as an accurate estimator of volatility. In strong contrast, RV is merely the sum of the squared intraday returns using high-frequency data. Ordinarily, to specify the dynamics of RV, time-series models are employed, including autoregressive fractionally integrated moving average (ARFIMA) and heterogeneous interval autoregressive (HAR) (Heterogeneous interval autoregressive, Corsi, 2009) models. In addition, an extension of both ARCH models and time-series models of RV is included in realized generalized ARCH (GARCH) models proposed by Hansen *et al.* (2012). These studies show that forecasts from daily RV estimates based on intraday returns are superior to forecasts from daily returns only.

There is an advantage to including RV in volatility models for forecasting. In the results of the previous research mentioned above, RV contains different information to that of daily returns, because RV is an observable volatility and is independent of the specification of volatility dynamics such as ARCH-type models. Accurately forecasting volatility is critical in option pricing.

In the mainstream option pricing literature, a wide range of traditional ARCH-type models are commonly analysed. Many of these models have already been applied to option pricing (Bollerslev and Mikkelsen (1999) and Duan 1995)). Some studies applied realized volatility to option pricing.

Bandi *et al.* (2008) apply certain types of RVs as described below to S&P 500 option prices, and conducted a trading simulation. Stentoft (2008) suggests a model that describes volatility dynamics using RV and latent volatility and shows it performs better for American companies' options. For HAR-type models, Corsi and Vecchia (2013), Jou *et al.* (2013), and Majewski *et al.* (2015) show that the model successfully predicts S&P 500 option prices. Ubukata and Watanabe (2014) predict Nikkei 225 put option prices using both ARFIMA(X) and HAR(X) models, and show that the ARFIMAX model performs best. Christoffersen *et al.* (2014) and Christoffersen *et al.* (2015) develop a new discrete-time model of volatility dynamics that contains both a GARCH component and an RV component, and find that RV reduces the S&P 500 index pricing errors of the benchmark model significantly. Therefore, this paper analyses whether realized GARCH models are useful for the pricing of Nikkei 225 options. The results indicate that realized GARCH models in this analysis perform better than either exponential GARCH (EGARCH) or Black-Scholes (BS) models in terms of option pricing.

Realized GARCH models have a number of advantages over both ARCH-type models and time-series models of RV. One advantage is that we can simultaneously estimate the dynamics of stock returns using both RV and daily return data. Another advantage is that we can adjust for the bias in RV caused by nontrading hours. Importantly, to my knowledge, relatively few studies have applied realized GARCH models to option pricing compared with applications to volatility forecasting. Accordingly, this paper applies realized GARCH models to the pricing of Nikkei 225 options traded at the Osaka Securities Exchange, and compares their performance with those using EGARCH and BS models.

As discussed, in actual markets, the presence of nontrading hours and market microstructure noise may cause bias in RV. Some available methods mitigate the effect of microstructure noise on RV, such as realized kernels (RK). We use RK proposed by Barndorff-Nielsen *et al.* (2008). For the bias associated with nontrading hours, we employ the bias-adjusted method proposed by Hansen and Lunde (2005). When using a log-linear specification, realized GARCH models can adjust the bias in RV in the same way as Hansen and Lunde (2005).

An alternative way of measuring volatility is the realized range (RR), which is based on the sum of the difference between the maximum and minimum prices observed during intraday intervals. Martens and van Dijk (2007) and Christensen and Podolskij (2007) show that RR is more efficient than the corresponding RV. Therefore, it is also important to estimate realized GARCH models with RR, and compare the results with those of RV.

Our main findings are as follows. In terms of option pricing, we find that realized GARCH models with RK with an adjustment for nontrading returns also perform better. This suggests that it is important to mitigate the microstructure noise on RV when we simulate option prices using realized GARCH models.

The remainder of the paper is structured as follows. Section 2 describes realized GARCH models. Section 3 describes the data used in the analysis and discusses integrated volatility and realized measures (RV, RK, and RR). In Section 4, we present the empirical results for realized GARCH models. Section 5 explains the method of calculating the option prices and compares the performance of the various option-pricing models in the analysis. Section 6 concludes.

2. Realized GARCH models

We begin with a brief review of realized GARCH models proposed by Hansen *et al.* (2012). Three equations characterize realized GARCH models; namely, the return equation, the GARCH equation, and the measurement equation. The return equation is specified as

$$r_t = E(r_t|F_{t-1}) + \varepsilon_t, \quad \varepsilon_t = \sqrt{h_t} z_t,$$
$$z_t \sim \text{i.i.d. } N(0,1), \qquad (1)$$

where r_t is the daily return on day t, h_t is the volatility of the daily return r_t, $E(r_t|F_{t-1})$ is the expectation of r_t conditional on the information available up to day $t-1$, and z_t is the standardized error, which follows an independent and identically distributed normal distribution with a mean of zero and a variance of one. In this analysis, the conditional expected return is specified as $E(r_t|F_{t-1}) = r + \nu_r\sqrt{h_t}$, where r is the risk-free rate and ν_r is a parameter of the risk premium. We specify this same equation not only for realized GARCH models but also for EGARCH models.

The second equation specified is the GARCH equation. We use the simplest version, log-realized GARCH(1, 1) model

$$\ln h_t = \omega + \beta \ln h_{t-1} + \gamma \ln x_{t-1}, \qquad (2)$$

where x_t is RV[1]. The error term for the return (r_t) affects latent volatility (h_t) through RV (x_{t-1}) in realized GARCH models.

The third equation specified is the measurement equation, where $\kappa(z_t)$ is known as the leverage function. This equation is specified as

$$\ln x_t = \xi + \phi \ln h_t + \kappa(z_t) + u_t,$$
$$u_t \sim \text{i.i.d. } N(0, \sigma_u^2), \qquad (3)$$
$$\kappa(z_t) = \kappa_1 z_t + \kappa_2(z_t^2 - 1). \qquad (4)$$

Given Eq.(3) and Eq.(4), RV (x_t) depends on the current value of z_t[2].

The leverage function $\kappa(z_t)$ expresses the volatility asymmetry. This reflects the well-known phenomenon in stock markets of a negative correlation between today's return and tomorrow's volatility. When $\kappa_1 < 0$ and $\gamma > 0$, the volatility asymmetry is observed.

We can derive the volatility persistence from the reduced form. More specifically, a realized GARCH (1, 1) model composed of Eq.(1), Eq.(2), and Eq.(3) implies a simple reduced-form model for $\{r_t, h_t\}$

$$\ln h_t = \mu_h + \pi \ln h_{t-1} + \gamma w_{t-1},$$
$$\ln x_t = \mu_x + \pi \ln x_{t-1} + w_t - \beta w_{t-1},$$

where $\pi = \beta + \phi\gamma$, $w_t = u_t + \kappa(z_t)$, $\mu_h = \omega + \gamma\xi$, $\mu_x = \phi\omega + (1-\beta)\xi$, and w_t is the error term in the measurement equation. The persistence of volatility is summarized by $\pi = \beta + \phi\gamma$. Thus, we can calculate the volatility persistence using both the GARCH equation and the measurement equation. In this model, volatility is stationary if $|\pi| < 1$.

Realized GARCH models can be estimated using quasi-maximum likelihood estimation. We adopt Gaussian specifications for the error terms u_t and z_t in the return and measurement equations, respectively, such that the log-likelihood function is given by

$$l(r, x; \theta) = -\frac{1}{2}\sum_{t=1}^{n}\left[\ln h_t + \frac{\varepsilon_t^2}{h_t} + \ln \sigma_u^2 + \frac{u_t^2}{\sigma_u^2}\right]. \qquad (5)$$

Here, θ is all of the parameters in realized GARCH models. See Hansen *et al.* (2012) for details.

While we assume that u_t and z_t follow normal distributions, it is well known that the distribution of stock returns is leptokurtic. In such a case, however, following the estimation of realized GARCH models, we cannot apply the Duan (1995) method to option pricing (Duan (1999)). To apply the Duan (1995) method to option pricing, we thus adopt a Gaussian specification for z_t.

3. Data

We employ Nikkei NEEDS-TICK data for estimating realized GARCH models and option-pricing simulations. The Japanese certificate of deposit (CD) rate serves as the risk-free rate. We now explain the method of data cleaning following Ubukata and Watanabe (2014), used for the closing prices of the Nikkei 225 stock index and the option prices.

The dataset comprises the Nikkei 225 stock index for each minute from 09 : 01 to 11 : 00 in the morning session (the closing time for the morning session has been extended from 11 : 00 to 11 : 30 since November 2011) and from 12 : 31 to 15 : 00 in the afternoon session. On occasion, the time stamps for the closing prices in the morning and afternoon sessions are slightly after 11 : 00 and 15 : 00, because the recorded time appears when the Nikkei 225 stock index is calculated. In such cases, we use all prices up to closing prices[3)].

Nikkei 225 options traded at the Osaka Securities Exchange are European options exercised only on the second Friday of each expiration month. For the most part, the options that have a maturity of 30 days (29 days if the month includes a holiday weekend) trade more heavily than other options with maturities shorter or longer than 30 days. In what follows, we concentrate on options with a maturity of 30 days. On such days, we consider options with different exercise prices whose bid and ask prices are both available at the same time between 14 : 00 to 15 : 00. For each option, we use the average of the bid and ask prices instead of the transaction prices because transaction prices are subject to market microstructure noise, as suggested by Campbell *et al.* (1997). We also exclude some kinds of put options not priced in the theoretical range from a lower bound at $P_T = \max(0, K\exp(-r\tau))$ to an upper bound at $P_T = K\exp(-r\tau)$. In sum, the numbers of call options and put options are 705 and 713. In more detail, the numbers of call options for the cases of $S_T/K < 0.91$, $0.91 \leq S_T/K < 0.97$, $0.97 \leq S_T/K < 1.03$, $1.03 \leq S_T/K < 1.09$ and, $1.09 \leq S_T/K$ are 197, 130, 114, 78 and 186, and, for put options, the numbers are 138, 99, 113, 100, and 263.

3. 1 Realized measures

We begin with a brief review of integrated volatility (IV) and RV using the following continuous price process. We assume that the log price process satisfies

$$dp(s) = \mu(s)ds + \sigma(s)dW(s), \quad (6)$$

where W is a standard Brownian motion, and μ and σ are smooth time-varying (random) functions. We let integer values of t correspond to the closing time of the afternoon session. The volatility over the interval $(t-1, t)$ is then defined as

$$IV_t = \int_{t-1}^{t} \sigma^2(s)ds. \quad (7)$$

We refer to this as IV for day t.

RV is an empirical estimate of IV constructed from intraday returns. For the special case where intraday returns are equidistant in calendar time, we define the intraday returns as

$$r(t-1+1/m), r(t-1+2/m), \cdots, r(t)$$

where m is the number of intraday returns. RV for day t is defined as the sum of squared intraday returns

$$RV_t = \sum_{i=1}^{m} r(t-1+i/m)^2 \quad (8)$$

RV_t will provide a consistent estimator of IV_t.

There are two problems in calculating RV: the first is the presence of nontrading hours, and the second is the presence of microstructure noise. We show that realized GARCH models are able to adjust for the bias associated with nontrading hours. We then detail the method used in Barndorff-Nielsen *et al.* (2008) for mitigating the effect of microstructure noise. Following this, we examine whether realized GARCH models using the bias-adjusted RV improve the option-pricing performance by comparing the results with those obtained using RV.

One problem in calculating RV is the presence of nontrading hours. To calculate RV that spans a full day, one also requires high-frequency data for the whole day. However, most equities trade for only a fraction of the day. For example, the Tokyo Stock Exchange is only open from 09 : 00 to 11 : 00 (morning session) and from 12 : 30 to 15 : 00 (afternoon session). Moreover, in Japan on the first and last trading days of the year, the market is only open from 09 : 00 to 11 : 00. In calculating RV using the above data, one may include returns on the nontrading hours, but this can make RV noisy because such returns include discretization noise. On the other hand, if we calculate RV as the sum of squared trading hours' returns only, RV may underestimate IV.

In terms of the bias associated with the presence of nontrading hours, (Hansen and Lunde (2005) consider a way to extend the RV_t, which is only available for trading hours, to a measure of volatility for the full day. Here, RVN_t indicates RV without nontrading hour returns. Their scaled estimator is

$$RVSC_t \equiv \hat{\delta} RVN_t, \quad \hat{\delta} = \frac{\sum_{t=1}^{n}(r_t - \bar{r})^2}{\sum_{t=1}^{n} RVN_t}, \quad (9)$$

where r_t is the daily returns, $\bar{r} = \frac{1}{n}\sum_{t=1}^{n} r_t$, and $\hat{\delta}$ is a consistent estimator of $\delta \equiv E[\sigma_t^2]/E[RVN_t]$. The mean of the $RVSC_t$ is equal to the volatility of daily returns. The abovementioned biases in RV (x_t) can be corrected with log-realized GARCH models, and we do not need to estimate δ.

The correcting bias in log-realized GARCH models is the same as the method of Hansen and Lunde (2005) in Eq.(9). When $x'_t = RVSC_t$ and $x_t = RVN_t$, a log realized GARCH model using x'_t is

$$\ln h_t = \omega + \beta \ln h_{t-1} + \gamma \ln x'_{t-1}, \quad (10)$$

$$\ln x'_t = \xi + \phi \ln h_t + \kappa(z_t) + u_t. \quad (11)$$

From $\ln x' = \ln \hat{\delta} + \ln x_t$,

$$\ln h_t = \omega + \gamma \ln \hat{\delta} + \beta \ln h_{t-1} + \gamma \ln x_{t-1}, \quad (12)$$

$$\ln x_t = \xi - \ln \hat{\delta} + \phi \ln h_t + \kappa(z_t) + u_t. \quad (13)$$

The constant estimates of $RVSC_t$ in Eq.(10) and Eq.(11) are different from RVN_t in Eq.(12) and Eq.(13), but other estimates of $RVSC_t$ are the same as those of RVN_t. Therefore, when we estimate log realized GARCH models, we do not need to calculate $RVSC_t$ and estimate them with RVN_t.

The other problem is the presence of microstructure noise (see Campbell *et al.* (1997), Ch. 3). When there is microstructure noise, market microstructure noise cause autocorrelation in intraday returns, and so RV includes not only the variance of the efficient price but also the variance of microstructure noise. If there is microstructure noise, RV becomes relatively large in the variance of the true return. That is, the bias caused by microstructure noise increases as the time interval approaches zero.

There are some methods available for mitigating the effect of microstructure noise on RV. The classic approach is to use RV constructed from intraday returns sampled at a moderate frequency. In practice, researchers are necessarily obliged to select a moderate sampling frequency. We calculate realized volatilities using 1-, 3-, and 5-minute intraday returns.

To mitigate the effect of microstructure noise, one of the kernel-based estimators is proposed by Barndorff-Nielsen *et al.* (2008). These estimators are called RK or flat-top kernels. Moreover, they compared the lower bound of parametric efficiency for some kernels, including the cubic, 5th to 8th order, Parzen, and modified Tukey-Hanning kernels. They concluded that only the modified Tukey-Hanning kernel, as detailed below, is more efficient than other kernels. Therefore, we focus on the modified Tukey-Hanning kernel estimator. We employ the flat-top

Tukey-Hanning kernel

$$TH_t = \hat{\gamma}_0 + \sum_{s=1}^{H} k(x)(\hat{\gamma}_s + \hat{\gamma}_{-s}), \quad x = \frac{s-1}{H},$$
$$k(x) = \sin^2\left\{\frac{\pi}{2}(1-x)^p\right\}, \qquad (14)$$
$$\hat{\gamma}_s = \sum_{j=1}^{m} r(t-1+j/m) \times r(t-1+(j-s)/m), \quad s = -H, \cdots, H.$$

Here, the nonstochastic $k(x)$ is a weight or kernel function, $\hat{\gamma}_0$ represents the RV, and $\hat{\gamma}_s$ represents the $s-$th autocovariance of the intraday returns. In this analysis, we set $p=2$ because it is nearly efficient and does not require too many intraday returns. Moreover, THN_t denotes the flat-top modified Tukey-Hanning kernel with $p=2$ without nontrading hour returns.

We estimate the asymptotically optimal value of H using 15-minute returns and the highest frequency 1-minute returns. See Barndorff-Nielsen *et al.* (2008) for details[4].

On the other hand, Christensen and Podolskij (2007) and Martens and van Dijk (2007) suggest RR be used to measure daily volatility by the sum of high-low ranges for intraday intervals. Christensen and Podolskij (2007) show that RR is unbiased and a consistent estimator of IV, and five times more efficient than the corresponding RV based on the same sampling frequency in an ideal world such as continuous trading with no market frictions.

RR for day t is defined as follows. First, Δ is defined for the $i-$th interval of length on day t, for $i=1, 2, \cdots, I$ with $I=1/\Delta$ assumed to be integer, and the price is observed m times during the $i-$th intraday interval. Then, we observe the high price $(H_{t,i})$ and the low price $(L_{t,i})$. We can aggregate high-low ranges for intraday intervals to obtain RR:

$$RR_t = \frac{1}{\lambda_m} \sum_{i=1}^{I} (\ln H_{t,i} - \ln L_{t,i})^2. \qquad (15)$$

Here, λ_m is the scaling factor, and is equal to the second moment of the range of a standard Brownian motion, $\lambda_m = \max_{0<s,t<m} \mathrm{E}(range_m^2)$ where

$$range_m = \sup_{0<s,t<m} (W_{t/m} - W_{s/m}). \qquad (16)$$

Moreover, λ_m converges to $4\ln(2)$ as m approaches ∞.

There are two problems to calculate RR. One problem is that there is no explicit formula of λ_m when m is a finite number. In fact, RR has a downward bias caused by the fact that the scaling factor $4\ln(2)$ is inappropriate because λ_m is an increasing function of m. Then, Christensen and Podolskij (2007) propose the simulation of λ_m when m is a finite number. The other problem is that RR may also be expected to suffer from market microstructure noise more than RV. Therefore, Martens and van Dijk (2007) consider a bias-adjustment procedure, which involves scaling RR with the ratio of the average level of the daily range and the average level of RR.

The above two methods are the same as the correcting bias in log-realized GARCH models. Both procedures can be written as

$$RRSC_t = \tilde{\delta} RR_t.$$

Here, $\tilde{\delta}$ is a constant, so that the form of this equation is the same as the method of Hansen and Lunde (2005). For the procedure by Christensen and Podolskij (2007), $\tilde{\delta} = (4\ln(2))/\lambda_{2,m}$. For the other procedure by Martens and van Dijk (2007) $\tilde{\delta} = \left(\frac{\sum_{l=1}^{q} RR_{t-1}^d}{\sum_{l=1}^{q} RR_{t-1}}\right)$ where RR_t^d denotes the daily range, and q is the number of the previous trading day. Therefore, we do not need to calculate $RRSC_t$.

We calculate RR using 3- and 5-minute intraday ranges with $\lambda_m = 4\ln(2)$. In addition, we cannot obtain prices for the nontrading hours, so that we denote this estimator RRN_t.

Table 1. Descriptive Statistics of Daily Data

		Mean	Std. Dev	Skewness	Kurtosis	Min	Max
r_t		0.012	1.341	−0.221	4.271	−6.864	5.735
RV_t	1-min.	1.145	0.850	2.073	12.819	0.084	9.647
	3-min.	1.310	0.965	1.956	10.984	0.085	9.829
	5-min.	1.352	0.987	1.855	9.906	0.082	9.078
RVN_t	1-min.	0.709	0.602	3.586	33.175	0.056	9.116
	3-min.	0.863	0.725	2.929	20.633	0.056	8.975
	5-min.	0.895	0.735	2.568	16.131	0.058	8.049
TH_t	1-min.	1.635	2.005	6.511	90.928	0.047	38.417
	3-min.	1.678	2.110	5.611	68.759	0.014	36.894
	5-min.	1.697	2.172	5.147	57.989	0.002	36.146
THN_t	1-min.	0.000	0.000	6.709	93.167	0.000	0.000
	3-min.	0.995	1.305	5.216	55.618	0.007	21.160
	5-min.	1.008	1.352	4.653	42.871	0.003	19.945
RRN_t	3-min.	0.487	0.403	16.779	16.394	0.034	4.719
	5-min.	0.792	0.668	18.382	19.649	0.058	7.690

Note) r_t denotes a daily stock return. RV_t denotes RV, RVN_t is RV without nontrading hour returns, TH_t is the flat-top Tukey-Hanning kernel with $p=2$ and THN_t is TH_t without nontrading hour returns. RRN_t is RR without nontrading hour returns. "1-min.," "3-min.," and "5-min." are the intraday return intervals used for calculating the volatility.

3. 2 Descriptive statistics for realized measures

Table 1 summarizes the descriptive statistics for daily return and the realized measures, RV, RK, and RR. First, as shown, all the means become larger as the sampling frequencies increase. This is contrary to our expectation that RV_t increases as the sampling frequency increases because of microstructure noise. Nonetheless, similar results arise in the volatility signature plots in Hansen and Lunde (2006) and Takahashi *et al.* (2009). Therefore, we consider that this phenomenon is because of the limited frequency available for our data. Second, all the standard deviations become larger as the time interval increases, and this confirms that intraday returns become noisy because of the discretization effect as the interval increases. These results suggest that a more precise estimator of the true volatility may be obtained by correcting the bias associated with nontrading hours and microstructure noise in RV_t. Third, the means of RVN_t and THN_t are relatively lower because RVN_t and THN_t are RV and RK only for trading hours.

4. Empirical results

We estimate realized GARCH models using 1,000 daily RV up to the day before the options trade. The estimated period is from May 2001 to September 2007 (77 months). The first options start trading on May 9, 2001. We first estimate the parameters in realized GARCH models using 1,000 daily RV, RK, RR, and returns up to May 8, 2001. We then repeat this procedure up to September 2007.

We first discuss the estimates of the parameters in the measurement equation. The persistence in volatility can be measured by the estimates of $\pi=\beta+\phi\gamma$. We find this is about 0.95, regardless of which realized measure is used. This result exhibits the well-known phenomenon of high persistence in volatility. Next, the asymmetry parameters κ_1 are estimated to be negative for RV_t, RVN_t, and RRN_t. This is also consistent with a well-known phenomenon in stock markets of a negative correlation between today's return and tomorrow's volatility, such as in Nelson (1991). However, the estimates of the asymmetry parameters κ_1 for TH_t and THN_t are not statistically significant and negative.

Table 2. Estimation Results 1 for 2007/09

		ν_t	ω	β	γ	$\log L$
1-min.	RV_t	0.048	0.190*	0.643*	0.420*	−427.964
		(0.032)	(0.036)	(0.061)	(0.077)	
	RVN_t	0.043	0.488*	0.529*	0.447*	−94.566
		(0.032)	(0.054)	(0.043)	(0.044)	
	TH_t	0.044	0.084*	0.797*	0.192*	−705.380
		(0.032)	(0.025)	(0.050)	(0.045)	
	THN_t	0.042	0.195*	0.781*	0.195*	−732.358
		(0.033)	(0.042)	(0.043)	(0.039)	
3-min.	RV_t	0.051	0.139*	0.626*	0.441*	−393.822
		(0.032)	(0.027)	(0.059)	(0.075)	
	RVN_t	0.042	0.388*	0.541*	0.431*	−181.456
		(0.032)	(0.047)	(0.046)	(0.045)	
	TH_t	0.041	0.085*	0.824*	0.162*	−875.565
		(0.032)	(0.027)	(0.049)	(0.043)	
	THN_t	0.040	0.163*	0.832*	0.146*	−951.781
		(0.032)	(0.038)	(0.035)	(0.032)	
	RRN_t	0.044	0.667*	0.524*	0.458*	−685.973
		(0.032)	(0.072)	(0.045)	(0.046)	
5-min.	RV_t	0.051	0.124*	0.629*	0.440*	−398.610
		(0.032)	(0.026)	(0.059)	(0.076)	
	RVN_t	0.043	0.358*	0.558*	0.418*	−222.531
		(0.032)	(0.047)	(0.046)	(0.047)	
	TH_t	0.040	0.087*	0.837*	0.147*	−969.193
		(0.034)	(0.027)	(0.045)	(0.039)	
	THN_t	0.039	0.159*	0.842*	0.134*	−1047.560
		(0.033)	(0.038)	(0.033)	(0.030)	
	RRN_t	0.043	0.442*	0.533*	0.454*	−637.773
		(0.032)	(0.052)	(0.046)	(0.045)	

Note) The numbers in parentheses are standard errors. * indicates significance at the 5% level.

Finally, the estimate for ν_r is only significant at the beginning of 2007. The implication is that there is only a risk premium in this period.

For example, Table 2 and Table 3 provide the empirical results for September 2007. As shown, the estimates of ξ are negative while those for ϕ are less than one. Consequently, all the realized measures exhibit downward biases. While κ_1 are estimated to be negative for RV_t, RVN_t, and RRN_t, the estimates of κ_1 are not significant for TH_t and THN_t.

From the results for ν_r, we can assume risk neutrality. We analyse discrete daily close-to-close returns, and we may represent the expected return under the assumption of risk neutrality by

$$r_t = r + \varepsilon_t,\quad \varepsilon_t = \sqrt{h_t} z_t,\quad z_t \sim \text{i.i.d.}\, N(0,1). \tag{17}$$

We estimate realized GARCH models using the return equation Eq.(17).

Under the assumption of risk neutrality, the persistence in volatility $\pi = \beta + \phi\gamma$ is estimated to be about 0.95. The estimates of the asymmetry parameters κ_1 are negative for RV_t, RVN_t, and RRN_t, but not significant for TH_t and THN_t. This result is the same as that without the assumption of risk neutrality. Table 4 and Table 5 provide the estimated results for September 2007 using the risk-neutral models.

5. Option pricing

Given the parameter estimates of realized GARCH models obtained, we now calculate the option prices. We begin with a brief review of option pricing using realized GARCH models and calculate option prices using the risk-neutral return equation in Eq. (17). We then explain the details of Duan (1995) in Section 5.1, and calculate option

Table 3. Estimation Results 2 for 2007/09

		ξ	ϕ	κ_1	κ_2	σ_u^2	π
1-min.	RV_t	−0.442*	0.702*	−0.106*	0.077*	0.299*	0.938
		(0.035)	(0.051)	(0.018)	(0.011)	(0.013)	
	RVN_t	−1.074*	0.938*	−0.095*	0.117*	0.143*	0.949
		(0.044)	(0.064)	(0.016)	(0.011)	(0.007)	
	TH_t	−0.404*	0.868*	0.061*	0.326*	0.476*	0.964
		(0.036)	(0.067)	(0.030)	(0.021)	(0.023)	
	THN_t	−0.964*	0.907*	−0.061*	0.222*	0.511*	0.958
		(0.043)	(0.076)	(0.028)	(0.019)	(0.023)	
3-min.	RV_t	−0.308*	0.710*	−0.113*	0.083*	0.281*	0.939
		(0.035)	(0.048)	(0.018)	(0.011)	(0.012)	
	RVN_t	−0.882*	0.947*	−0.110*	0.116*	0.171*	0.949
		(0.044)	(0.064)	(0.016)	(0.011)	(0.008)	
	TH_t	−0.482*	0.848*	0.089	0.389*	0.661*	0.961
		(0.036)	(0.070)	(0.035)	(0.024)	(0.037)	
	THN_t	−1.071*	0.883*	−0.042	0.284*	0.785*	0.961
		(0.043)	(0.081)	(0.035)	(0.023)	(0.037)	
	RRN_t	−1.439*	0.907*	−0.117*	0.091*	0.181*	0.940
		(0.044)	(0.061)	(0.016)	(0.011)	(0.009)	
5-min.	RV_t	−0.276*	0.706*	−0.108	0.080*	0.284*	0.940
		(0.035)	(0.048)	(0.018)	(0.011)	(0.013)	
	RVN_t	−0.842*	0.938*	−0.106*	0.107*	0.187*	0.950
		(0.044)	(0.062)	(0.017)	(0.010)	(0.009)	
	TH_t	−0.535*	0.825*	0.107	0.429*	0.792*	0.958
		(0.036)	(0.072)	(0.040)	(0.025)	(0.051)	
	THN_t	−1.135*	0.857*	−0.031	0.323*	0.946*	0.957
		(0.043)	(0.084)	(0.039)	(0.025)	(0.045)	
	RRN_t	−0.958*	0.917*	−0.106*	0.116*	0.167*	0.949
		(0.042)	(0.055)	(0.016)	(0.011)	(0.008)	

Note) The numbers in parentheses are standard errors. * indicates significance at the 5% level.

prices using Eq. (1).

The price of a European option is equal to the discounted present value of the expectation of option prices on the expiration date. For example, the prices of European call and put options, with the exercise price K and survival period τ, are given by

$$C_T = \left(\frac{1}{1+r}\right)^{\tau} E^Q[\max(\tilde{S}_{T+\tau}-K, 0)],$$
$$P_T = \left(\frac{1}{1+r}\right)^{\tau} E^Q[\max(K-\tilde{S}_{T+\tau}, 0)]. \tag{18}$$

Here, $\tilde{S}_{T+\tau}$ is the price of the underlying asset on the expiration date $T+\tau$.

We cannot evaluate this expectation analytically if the volatility of the underlying asset follows realized GARCH models. We instead calculate this expectation by simulating $\tilde{S}_{T+\tau}$ from realized GARCH models.

The simulation procedure is as follows. First, we set the parameters of realized GARCH models equal to their estimates. Next, we generate random values of z_t and u_t, and substitute them, $z_{T+1}, \cdots, z_{T+\tau}$ and $u_{T+1}, \cdots, u_{T+\tau}$, into realized GARCH models to obtain $(S_{T+1}^{(1)}, \cdots, S_{T+\tau}^{(1)})$. After that, we repeat this procedure once. Suppose that $(S_{T+\tau}^{(1)}, \cdots, S_{T+\tau}^{(I)})$ are simulated. Then, for variance reduction, we use the control variates and antithetic variates jointly. See the appendix for details.

For comparison, we also calculate option prices using EGARCH(1, 0) models (Nelson (1991)) and the well-known BS (Black and Scholes (1973)) models with volatility as the standard deviation of daily returns over the past 20 days.

To measure the option-pricing performance, we use the root mean squared error ratio defined by

Tabel 4. Estimation Results 1 for 2007/09 (risk neutral)

		ν_t	ω	β	γ	$\log L$
1-min.	RV_t	——	0.191*	0.642*	0.421*	−429.128
			(0.036)	(0.061)	(0.077)	
	RVN_t	——	0.489*	0.529*	0.448*	−95.472
			(0.054)	(0.043)	(0.044)	
	TH_t	——	0.084*	0.796*	0.193*	−706.361
			(0.025)	(0.049)	(0.045)	
	THN_t	——	0.196*	0.781*	0.196*	−733.230
			(0.042)	(0.044)	(0.039)	
3-min.	RV_t	——	0.140*	0.625*	0.442*	−394.948
			(0.027)	(0.059)	(0.075)	
	RVN_t	——	0.389*	0.541*	0.431*	−182.333
			(0.047)	(0.046)	(0.045)	
	TH_t	——	0.086*	0.824*	0.162*	−876.421
			(0.027)	(0.048)	(0.043)	
	THN_t	——	0.164*	0.831*	0.147*	−952.592
			(0.038)	(0.035)	(0.032)	
	RRN_t	——	0.668*	0.524*	0.458*	−688.030
			(0.072)	(0.045)	(0.046)	
5-min.	RV_t	——	0.125*	0.628*	0.441*	−399.757
			(0.026)	(0.059)	(0.076)	
	RVN_t	——	0.359*	0.558*	0.418*	−223.461
			(0.047)	(0.046)	(0.047)	
	TH_t	——	0.087*	0.836*	0.148*	−969.989
			(0.027)	(0.045)	(0.039)	
	THN_t	——	0.160*	0.842*	0.134*	−1048.332
			(0.038)	(0.034)	(0.030)	
	RRN_t	——	0.443*	0.533*	0.454*	−639.848
			(0.052)	(0.046)	(0.045)	

Noter) The numbers in parentheses are standard errors. * indicates significance at the 5% level.

$$RMSER = \begin{cases} \sqrt{\frac{1}{N_C}\sum_{i=1}^{N_C}(\tilde{C}_i - C_i)^2/C_i}, & \text{for call options,} \\ \sqrt{\frac{1}{N_P}\sum_{i=1}^{N_P}(\tilde{P}_i - P_i)^2/P_i}, & \text{for put options.} \end{cases}$$

Here N_C and N_P are the numbers of call and put options, and $\tilde{C}_i$ and $\tilde{P}_i$ are the price of the i−th call and put option calculated from realized GARCH, EGARCH, or BS models. C_i and P_i are the market call and put price.

From the results of option pricing under the assumption of risk neutrality in Table 6, we can see that the RMSER of realized GARCH models with THN_t and RV_t for the put options and that of EGARCH models for the call options are smaller than the RMSER of the other models. These results depend on which realized measures we use. Therefore, we compare the RMSER of the different realized measures.

First, the RMSER of realized GARCH models with TH_t is smaller than that of those with RV_t except for the RMSER of put options using 1-minute and 5-minute intraday returns. The difference between RV_t and TH_t is the presence of microstructure noise. For TH_t, in calculation of the flat-top Tukey-Hanning kernel, the effect of microstructure noise is corrected. As a result, it is important to mitigate the microstructure noise on RV when we simulate option prices using realized GARCH models.

Next, the RMSER of realized GARCH models with THN_t is smaller than that of those with TH_t. THN_t does not include the nontrading hour returns, and, as mentioned above, the correcting bias of log-realized GARCH models is the same as the method of Hansen and Lunde (2005) because we take the log of x_t. In sum, THN_t is a more accurate

Table 5. Estimation Results 2 for 2007/09 (risk neutral)

		ξ	ϕ	κ_1	κ_2	σ_u^2	π
1-min.	RV_t	−0.438*	0.702*	−0.113*	0.078*	0.299*	0.938
		(0.035)	(0.051)	(0.017)	(0.011)	(0.013)	
	RVN_t	−1.071*	0.937*	−0.106*	0.117*	0.143*	0.949
		(0.043)	(0.064)	(0.015)	(0.011)	(0.007)	
	TH_t	−0.407*	0.868*	0.032	0.326*	0.476*	0.964
		(0.037)	(0.067)	(0.043)	(0.021)	(0.024)	
	THN_t	−0.962*	0.906*	−0.080*	0.222*	0.511*	0.958
		(0.043)	(0.075)	(0.026)	(0.019)	(0.023)	
3-min.	RV_t	−0.304*	0.710*	−0.121*	0.083*	0.281*	0.939
		(0.035)	(0.048)	(0.018)	(0.011)	(0.012)	
	RVN_t	−0.878*	0.946*	−0.120*	0.116*	0.171*	0.949
		(0.044)	(0.063)	(0.017)	(0.011)	(0.008)	
	TH_t	−0.485*	0.848*	0.056	0.389*	0.661*	0.961
		(0.037)	(0.070)	(0.035)	(0.024)	(0.037)	
	THN_t	−1.070*	0.882*	−0.065	0.284*	0.785*	0.960
		(0.043)	(0.080)	(0.032)	(0.023)	(0.037)	
	RRN_t	−1.436*	0.906*	−0.125*	0.091*	0.181*	0.940
		(0.044)	(0.061)	(0.016)	(0.011)	(0.009)	
5-min.	RV_t	−0.272*	0.706*	−0.116*	0.080*	0.285*	0.939
		(0.034)	(0.048)	(0.018)	(0.011)	(0.013)	
	RVN_t	−0.838*	0.938*	−0.116*	0.107*	0.187*	0.950
		(0.044)	(0.062)	(0.016)	(0.010)	(0.009)	
	TH_t	−0.539*	0.825*	0.072	0.430*	0.792*	0.958
		(0.038)	(0.073)	(0.108)	(0.025)	(0.051)	
	THN_t	−1.134*	0.857*	−0.057	0.324*	0.946*	0.957
		(0.043)	(0.084)	(0.031)	(0.025)	(0.045)	
	RRN_t	−0.955*	0.916*	−0.116*	0.116*	0.167*	0.949
		(0.042)	(0.055)	(0.016)	(0.012)	(0.008)	

Note) The numbers in parentheses are standard errors. * indicates significance at the 5% level.

Table 6. RMSER for Option Pricing (risk neutral)

	CALL			PUT		
BS	7.830			2.587		
EG	1.914*			1.539		
RGAR	1-min.	3-min.	5-min.	1-min.	3-min.	5-min.
RV_t	3.769	3.923	3.710	**1.253**	**1.422**	**1.233**
RVN_t	4.379	4.341	3.993	1.627	1.619	**1.436**
TH_t	3.742	3.459	3.389	1.688	**1.375**	**1.269**
THN_t	2.771	2.595	2.610	**0.828***	**0.650***	**0.630***
RRN_t		4.014	4.431		**1.528**	1.545

Note) * indicates the smallest RMSER. The bold numbers mean that its RMSER is smaller than that of EGARCH model.

volatility measure than TH_t. Although RVN_t and RRN_t do not include the nontrading hour returns, as is also the case for THN_t, they are affected by microstructure noise.

Moreover, the RMSER of realized GARCH models with RRN_t is not smaller than that of those with RVN_t. Theoretically, RRN_t is five times more efficient, but is affected by the microstructure noise more than RVN_t. That is, from this result, the RRN_t of these data is not a more efficient estimator than RVN_t, because of the microstructure noise.

In Table 7, for the case of S_T/K less than 1.03, the RMSER of realized GARCH models with THN_t is smaller than that of EGARCH models except for the RMSER of the call options using 1-minute intraday returns. Consequently, realized GARCH models perform better with THN_t than with RV_t, RVN_t, TH_t, and RRN_t.

Finally, the best-performing models are realized GARCH models with THN_t for put options, with more accurate volatility than

Table 7. RMSE for Option Pricing by Moneyness (risk neutral)

	S_T/K	CALL			PUT		
BS	・<0.91	13.454			0.024		
	0.19≤・<0.97	7.572			0.181		
	0.97≤・<1.03	0.961			0.806		
	1.03≤・<1.09	0.158			2.537		
	1.09≤・	0.031			3.925		
EG	・<0.91	3.298			0.020		
	0.19≤・<0.97	1.812			0.087		
	0.97≤・<1.03	0.313			0.319		
	1.03≤・<1.09	0.077			1.039		
	1.09≤・	0.026			2.442		
RGAR	S_t/K	1-min.	3-min.	5-min.	1-min.	3-min.	5-min.
RV_t	・<0.91	6.632	6.927	6.584	0.021	0.021	0.021
	0.19≤・<0.97	3.199	3.259	2.972	0.104	0.106	0.100
	0.97≤・<1.03	0.395	0.396	0.380	0.375	0.380	0.344
	1.03≤・<1.09	0.078	0.080	0.078	1.207	1.271	1.083
	1.09≤・	**0.026**	**0.026**	**0.026**	**1.907**	**2.192**	**1.902**
RVN_t	・<0.91	7.666	7.679	7.065	0.021	0.021	0.021
	0.19≤・<0.97	3.837	3.554	3.262	0.114	0.107	0.104
	0.97≤・<1.03	0.463	0.449	0.430	0.453	0.411	0.384
	1.03≤・<1.09	0.088	0.087	0.085	1.498	1.359	1.217
	1.09≤・	0.026	0.027	0.027	2.496	2.515	**2.227**
TH_t	・<0.91	6.402	5.989	5.844	0.021	0.021	0.021
	0.19≤・<0.97	3.702	3.227	3.231	0.115	0.107	0.108
	0.97≤・<1.03	0.362	0.340	0.343	0.426	0.379	0.384
	1.03≤・<1.09	**0.074**	**0.069**	**0.069**	1.558	1.290	1.289
	1.09≤・	**0.026**	**0.025**	**0.025**	2.593	**2.104**	**1.914**
THN_t	・<0.91	4.844	4.568	4.589	0.021	0.021	0.021
	0.19≤・<0.97	2.446	2.194	2.225	0.089	**0.084**	**0.084**
	0.97≤・<1.03	0.332	**0.296**	**0.293**	**0.300**	**0.268**	**0.264**
	1.03≤・<1.09	**0.070**	**0.063**	**0.062**	**0.850**	**0.721**	**0.696**
	1.09≤・	0.026	**0.026**	**0.026**	**1.241**	**0.956**	**0.927**
RRN_t	・<0.91		7.080	7.859		0.021	0.021
	0.19≤・<0.97		3.348	3.562		0.102	0.105
	0.97≤・<1.03		0.447	0.457		0.397	0.401
	1.03≤・<1.09		0.086	0.090		1.261	1.293
	1.09≤・		0.027	0.027		**2.378**	**2.400**

Note) The bold numbers mean that its RMSE is smaller than that of EGARCH model.

that from other models for the following reasons. First, in calculation of the flat-top Tukey-Hanning kernel, the effect of microstructure noise is corrected. Second, THN_t does not include the nontrading hour returns. Then, taking the log of x_t in realized GARCH models, the bias is corrected using the Hansen and Lunde (2005) method. On the other hand, the best-performing models for put options are EGARCH models, but the smallest RMSER of realized GARCH models is the RMSER with THN_t.

5.1 Duan convert

Regarding the option pricing, unless traders are risk neutral, we must convert the physical measure P into the risk-neutral measure Q. After converting the models, we evaluate the option prices under the risk-neutral measure Q.

Duan (1995) makes the following assumptions on Q, called the local risk-neutral valuation relationship (LRNVR):

- $r_t|F_{t-1}$ follows a normal distribution under the risk-neutral measure Q,
- $E^Q[r_t|F_{t-1}] = r$,
- $Var^Q[r_t|F_{t-1}] = Var^P[r_t|F_{t-1}]$.

For realized GARCH models, because z_t follows a standard normal distribution, the conditional return under the physical meas-

Table 8. RMSER for Option Pricing

	CALL			PUT		
BS	7.830			2.587		
EG	2.011*			1.568		
RGAR	1-min.	3-min.	5-min.	1-min.	3-min.	5-min.
RV_t	3.639	3.826	3.722	**1.130**	**1.319**	**1.274**
RVN_t	4.367	4.381	4.032	**1.460**	**1.505**	**1.437**
TH_t	3.571	3.353	3.304	**1.380**	**0.903**	**1.122**
THN_t	2.841	2.564	2.611	**0.805***	**0.642***	**0.624***
RRN_t		3.862	4.321		**1.519**	1.582

Note) * indicates the smallest RMSER. The bold numbers mean that its RMSER is smaller than that of EGARCH model.

ure P follows

$$r_t|F_{t-1} \sim N(E(r_t|F_{t-1}), h_t|F_{t-1}),$$

where the mean of conditional return $E(r_t|F_{t-1})$ and the variance $h_t|F_{t-1}$ are nonstochastic variables. Thus, the Duan (1995) method can be applied to realized GARCH models.

Under the assumptions of LRNVR, daily returns under the risk-neutral measure Q are represented by

$$r_t^Q = r+\eta_t, \quad \eta_t \sim \text{i.i.d.}\, N(0, h_t), \tag{19}$$

$$\varepsilon_t^Q = \eta_t + r - E(r_t|F_{t-1}), \tag{20}$$

$$z_t^Q = \frac{\varepsilon_t^Q}{\sqrt{h_t}}. \tag{21}$$

In this study, $E(r_t|F_{t-1}) = \nu_r\sqrt{h_t}$. All we have to do for volatility is to substitute z_t^Q in realized GARCH models.

From the results using the Duan (1995) method in Tabel 8, for put options, the RMSER for realized GARCH models is smaller than that for EGARCH or BS models except for TH_t using 5-minute intraday returns, and for call options, the RMSER for realized GARCH models is not smaller than that for EGARCH models. The RMSER of realized GARCH models with TH_t is smaller than that of those with RV_t except for TH_t using 1-minute intraday returns for put options. Next, realized GARCH models with RVN_t do not perform better than RV_t, and realized GARCH models with THN_t perform better than RV_t, RVN_t, TH_t, and RRN_t. These results are consistent with the results under the assumption of risk neutrality. In Table 9, in the case of S_T/K less than 1.03, the RMSER of realized GARCH models with THN_t is smaller than that of EGARCH models except for the RMSER for call options using 1-minute intraday returns. Consequently, realized GARCH models perform better with THN_t than with RV_t, RVN_t, TH_t, and RRN_t.

In addition, we compare these results with the results in Table 6. The RMSER using the Duan (1995) method in Table 8 is smaller than that under the assumption of risk neutrality in Table 6, except for put options, RV_t, RNV_t, and RRN_t using 5-minute intraday returns, and for call options, RV_t using 5-minute intraday returns, RVN_t using 3-minute and 5-minute intraday returns, and THN_t using 5-minute intraday returns. Although the risk parameter ν_t is not significant in the estimation results of realized GARCH models, we do not set the risk parameter ν_t equal to zero when we simulate the option prices. Therefore, the simulated option prices are different from those under the risk-neutral assumption. This means that the Duan (1995) method improves pricing performance, even though the estimate of the risk-premium parameter is not significant.

6. Conclusions

This paper compares the option-pricing performance using realized GARCH and EGARCH models. The main results are as follows. First, from the results for put options

Table 9. RMSER for Option Pricing by Moneyness

	S_T/K	CALL			PUT		
BS	・<0.91	13.454			0.024		
	0.19≤・<0.97	7.572			0.181		
	0.97≤・<1.03	0.961			0.806		
	1.03≤・<1.09	0.158			2.537		
	1.09≤・	0.031			3.925		
EG	・<0.91	3.504			0.020		
	0.19≤・<0.97	1.801			0.087		
	0.97≤・<1.03	0.313			0.318		
	1.03≤・<1.09	0.078			1.028		
	1.09≤・	0.026			2.494		
RGAR	S_T/K	1-min.	3-min.	5-min.	1-min.	3-min.	5-min.
RV_t	・<0.91	6.422	6.792	6.624	0.020	0.021	0.021
	0.19≤・<0.97	3.029	3.055	2.918	0.099	0.100	0.100
	0.97≤・<1.03	0.391	0.393	0.382	0.347	0.353	0.344
	1.03≤・<1.09	**0.077**	0.079	**0.077**	1.056	1.125	1.088
	1.09≤・	**0.026**	0.026	**0.026**	**1.726**	**2.044**	**1.974**
RVN_t	・<0.91	7.731	7.801	7.144	0.021	0.021	0.021
	0.19≤・<0.97	3.560	3.417	3.266	0.105	0.105	0.104
	0.97≤・<1.03	0.462	0.443	0.430	0.407	0.394	0.384
	1.03≤・<1.09	0.086	0.086	0.085	1.272	1.254	1.226
	1.09≤・	0.027	0.027	0.026	**2.255**	**2.339**	**2.227**
TH_t	・<0.91	6.179	5.909	5.741	0.021	0.021	0.021
	0.19≤・<0.97	3.345	2.821	3.028	0.110	0.097	0.105
	0.97≤・<1.03	0.356	0.332	0.338	0.399	0.320	0.368
	1.03≤・<1.09	**0.071**	**0.067**	**0.068**	1.363	**0.913**	1.180
	1.09≤・	**0.025**	**0.025**	**0.025**	**2.093**	**1.359**	**1.679**
THN_t	・<0.91	4.991	4.520	4.609	0.020	0.021	0.020
	0.19≤・<0.97	2.432	2.146	2.168	0.088	**0.084**	**0.084**
	0.97≤・<1.03	0.330	**0.294**	**0.291**	**0.297**	**0.266**	**0.264**
	1.03≤・<1.09	**0.070**	**0.063**	**0.062**	**0.844**	**0.709**	**0.695**
	1.09≤・	0.026	**0.026**	**0.025**	**1.202**	**0.946**	**0.917**
RRN_t	・<0.91		6.803	7.641		0.021	0.020
	0.19≤・<0.97		3.252	3.552		0.102	0.104
	0.97≤・<1.03		0.446	0.451		0.394	0.400
	1.03≤・<1.09		0.087	0.089		1.246	1.291
	1.09≤・		0.027	0.027		**2.366**	**2.465**

Note) The bold numbers mean that its RMSER is smaller than that of EGARCH model.

assuming risk neutrality, realized GARCH models with RK and RV perform better than either EGARCH or BS models. However, for call options, realized GARCH models do not improve the results for RMSER. Without the assumptions of risk neutrality, for put options, realized GARCH models with RK perform better than EGARCH and BS models; however, for call options, EGARCH models perform better than the other models.

Irrespective of the risk-neutrality assumption, for put options, the best-performing models are realized GARCH models with THN_t (the flat-top Tukey-Hanning kernel method without the lunch-time and overnight returns). From these results, we can see that the flat-top Tukey-Hanning kernel method improves option-pricing performance. Therefore, option-pricing performance improves when using accurate estimators of IV.

Several extensions are possible. First, we assume the risk-neutral volatility dynamics are the same as the physical dynamics. However, Corsi *et al.* (2009), Christoffersen *et al.* (2014), and Christoffersen *et al.* (2010) propose option-pricing methods when the risk-neutral volatility dynamics differ from the physical volatility dynamics. Barone-Adei *et al.* (2008) propose a method for pricing options that allows for different distributions (volatilities) under the physical measure P and the risk-neutral measure Q. These methods can be adapted for realized GARCH models. Second, we did not consider jumps in intraday returns. Barndorff-Nielsen and Shepard (2004) and Dobrev and Szerszen (2010) have proposed a method to calculate RV taking jumps into account. It would be interesting to see whether the option-pricing performance improves using these realized measures. Third, we only analyse short-term options in this paper; we should also simulate and analyse long-term options. Finally, Takahashi *et al.* (2009), Dobrev and Szerszen (2010), and Koopman and Scharth (2011) propose realized stochastic volatility models that have similar advantages as realized GARCH models. These could be compared with the performance of realized GARCH models.

A. Call and Put Option Prices Using Realized GARCH Models

We calculate the call and put option prices (C_T, P_T) as follows.

1. We set the parameters of realized GARCH models equal to their estimates.
2. We generate random values for z_t and u_t, and substitute $(z_{T+1}, \cdots, z_{T+\tau})$ and $(u_{T+1}, \cdots, u_{T+\tau})$, into realized GARCH models to obtain $(S_{T+1}^{(1)}, \cdots, S_{T+\tau}^{(1)})$.
$$r_t = E(r_t|F_{t-1})+\varepsilon_t, \quad \varepsilon_t = \sqrt{h_t}z_t, \tag{22}$$
$$\ln h_t = \omega+\beta \ln h_{t-1}+\gamma \ln x_{t-1},$$
$$\ln x_{t-1} = \xi+\phi \ln h_{t-1}+\kappa(z_{t-1})+u_{t-1},$$
$$\kappa(z_{t-1}) = \kappa_1 z_{t-1}+\kappa_2(z_{t-1}^2-1).$$
3. We substitute $(-z_{T+1}, \cdots, -z_{T+\tau})$ and $(-u_{T+1}, \cdots, -u_{T+\tau})$, into realized GARCH models to obtain $(S_{T+1}^{(2)}, \cdots, S_{T+\tau}^{(2)})$, again.
4. We simulate BS models by substituting the same values of $(z_{T+1}, \cdots, z_{T+\tau})$ into
$$r_t = r-\frac{1}{2}\sigma^2+\varepsilon_t, \tag{23}$$
$$\varepsilon_t = \sigma z_t. \tag{24}$$
Here, σ is the volatility measured as the standard deviation of daily returns over the past 20 days. We can obtain the values of $(S_{T+1}^{BS(1)}, \cdots, S_{T+\tau}^{BS(1)})$. Then, we simulate BS models using the same values of $(-z_{T+1}, \cdots, -z_{T+\tau})$ to obtain $(S_{T+1}^{BS(2)}, \cdots, S_{T+\tau}^{BS(2)})$.
5. We repeat the above procedures $l/2$ times. Suppose that $(S_{T+1}^{(1)}, \cdots, S_{T+\tau}^{(l)})$ and $(S_{T+\tau}^{BS(1)}, \cdots, S_{T+\tau}^{BS(l)})$ are simulated. In this paper, we set $l=10{,}000$.
6. We can calculate the option prices that allow us to substitute $(S_{T+1}^{(1)}, \cdots, S_{T+\tau}^{(l)})$ into the below equations.
$$C_T^{(i)} = \left(\frac{1}{1+r}\right)^\tau \max(S_{T+\tau}^{(i)}-K, 0),$$
$$i = 1, \cdots, l,$$
$$P_T^{(i)} = \left(\frac{1}{1+r}\right)^\tau \max(K-S_{T+\tau}^{(i)}, 0),$$
$$i = 1, \cdots, l. \tag{25}$$
Moreover, we substitute $(S_{T+\tau}^{BS(1)}, \cdots, S_{T+\tau}^{BS(l)})$ into
$$C_T^{BS(i)} = \exp(-r\tau)\times \max(S_{T+\tau}^{BS(i)}-K, 0), \quad i = 1, \cdots, l,$$
$$P_T^{BS(i)} = \exp(-r\tau)\times \max(K-S_{T+\tau}^{BS(i)}, 0), \quad i = 1, \cdots, l. \tag{26}$$
Finally, we obtain $(C_T^{(1)}, \cdots, C_T^{(l)})$, $(C_T^{BS(1)}, \cdots, C_T^{BS(l)})$, $(P_T^{(1)}, \cdots, P_T^{(l)})$ and $(P_T^{BS(1)}, \cdots, P_T^{BS(l)})$.
7. Using the above-simulated option prices, we calculate
$$\phi_C = \frac{\mathrm{Cov}(C_T^{(\cdot)}, C_T^{BS(\cdot)})}{\mathrm{Var}(C)}, \tag{27}$$
$$\phi_P = \frac{\mathrm{Cov}(P_T^{(\cdot)}, P_T^{BS(\cdot)})}{\mathrm{Var}(P)}. \tag{28}$$
Here, $C_T^{(\cdot)}=\{C_T^{(1)}, \cdots, C_T^{(l)}\}$, $C_T^{BS(\cdot)}=\{C_T^{BS(1)}, \cdots, C_T^{BS(l)}\}$, $P_T^{(\cdot)}=\{P_T^{(1)}, \cdots, P_T^{(l)}\}$ and $P_T^{BS(\cdot)}=\{P_T^{BS(1)}, \cdots, P_T^{BS(l)}\}$. Then, we calculate
$$\widehat{C}_T^{(i)} = \phi_C C_T^{BS(i)}+(C_T^{(i)}-\phi_C C_T^{BS}),$$
$$i = 1, \cdots, l, \tag{29}$$
$$\widehat{P}_T^{(i)} = \phi_P P_T^{BS(i)}+(P_T^{(i)}-\phi_P P_T^{BS}),$$
$$i = 1, \cdots, l. \tag{30}$$
Here, C_T^{BS} and P_T^{BS} are the option prices using BS (Black and Scholes (1973)) models with volatility as the standard deviation of daily returns over the past 20. We obtain $(\widehat{C}_T^{(1)}, \cdots, \widehat{C}_T^{(T)})$ and $(\widehat{P}_T^{(1)}, \cdots, \widehat{P}_T^{(T)})$.
8. Then, Eq.(18) can be calculated as the

average of $(\widehat{C}_T^{(1)}, \cdots, \widehat{C}_T^{(l)})$ or $(\widehat{P}_T^{(1)}, \cdots, \widehat{P}_T^{(l)})$.

$$\widetilde{C}_T = \frac{1}{l}\sum_i \widehat{C}_T^{(i)}, \quad \widetilde{P}_T = \frac{1}{l}\sum_i \widehat{P}_T^{(i)}. \tag{31}$$

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Notes

1) Generally, the realized GARCH (p, q) model replaces Eq.(2) with

$$\ln h_t = \omega + \sum_{i=1}^{p}\beta_i \ln h_{t-i} + \sum_{j=1}^{q}\gamma_j \ln x_{t-j}.$$

We estimate only a realized GARCH(1, 1) model.

2) Hansen *et al.* (2012) consider leverage functions that are constructed from Hermite polynomials

$$\kappa(z) = \kappa_1 z + \kappa_2(z^2-1) + \kappa_3(z^3-3z) + \kappa_4(z^4-6z^2+3) + \cdots,$$

and $\kappa(z_t) = \kappa_1 z_t + \kappa_2(z_t^2-1)$.

3) In their analysis, Hansen and Lunde (2005) use intraday returns constructed for both bid and ask prices using the previous-tick interpolation method. We define the overnight return as the log difference between the first price (mid quote) of the day and the last price (mid quote) of the preceding day.

4) From an empirical perspective, Barndorff-Nielsen et al (2008) point out that end effects can be safely ignored in practice, despite their important theoretical implications for the asymptotic properties of RK estimators. Thus, we use all samples to calculate RK.

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