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Feasible Nash Implementation of Competitive Equilibria in an Economy with Externalities

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1. Introduction

In this paper we consider the possibility of Nash implementation of competitive equilibria in an economy with externalities. The issue of incentive compatibility in economies with externalities has three independent bodies of literature as its background. One is the Nash implementation of Walras and/or Lindahl equilibria. Another body of literature analyzes the definition and characterisitics of competitive equilibria in economies with externalities. The third is the Nash implementation of some target correspondence in the context of social choice. This paper is an attempt to synthesize these three approaches, which are now discussed briefly in turn.

Incentive compatibility itself was discussed originally in the context of a public goods economy. Samuelson (1954) was the first to address the "free rider problem" in Lindahl equilibria. He pointed out that we cannot expect Lindahl equiliria to be achieved if rational agents behaved strategically. In 1972, Hurwicz (1972) showed that this kind of problem could occur even in the standard Walrasian economy. Numerous contributions followed concerning Nash implementation of both Walras and Lindahl equilibria, including those of Schmeidler (1980), Hurwicz (1979b) and Walker (1981). These papers propose mechanisms which implement eiter Walrasian or Lindahl equilibrium, which also satisfy the condition of demand equals supply even outside equilibrium. Jordan (1982) found a general method of modifying these mechanisms to include production. However, none of these papers consider individual feasibility, namely the requirement that the outcome belong to each individual's consumption set even outside equilibrium. In contrast, Hurwicz, Maskin and Postlewaite (1984), Postlewaite and Wettstein (1989) and Tian (1989) construct mechanisms satisfying, in addition to the demand and supply balance condition, the individual feasibility condition. As far as stability is concerned, Jordan (1986) proves the non-existence of mechanisms which implement Walrasian equilibrium and are dynamically stable in classical environments. Kim (1987) proves the same impossibility theorem in an economy with public goods. He also constructs a mechanism which implements Lindahl equilibrium and is dynamically stable if we restrict the environment of allow only quasi-linear utility functions.

On the other hand, Aoki (1971) discusses the relation between competitive equilibria and Pareto optimal allocations in an economy with externalities. His analysis is restricted to an economy of a very special type in which there is only a single consumer and externalities exist only within each industry. In a more general framework, Osana (1977) shows the existence of equilibria and proves that every Pareto optimal allocation is a competitive equilibrium if some suitable tax-subsidy system is adopted. This is even without transfers of individual endowments. The closest equilibrium concept to the one used in this paper is that of Otani and Sicilian (1977). The conditions for their equilibrium is stronger than those of Osana (1977). They prove the first and second fundamental theorems of welfare economics using their definition of competitive equilibria, but do not prove the existence of such equilibria. None of the above studies consider implementability of the competitive equilibrium.

As for the third body of literature, Hurwicz and Schmeidler (1978) construct some mechanisms guaranteeing the existence of Nash equilibrium and the Pareto optimality of the equilibrium, when the set of alternatives is finite. Saijo (1988) proves Maskin (1977)'s theorem which states necessary conditions as well as sufficient conditions for the mechanisms to implement any given target correspondence in a general social choice framework. In such mechanisms, however, each agent must know the socially attainable set. Furthermore, exchange of messages between agents may be difficult, since their individual message spaces have infinite dimensions.

This paper shows that a competitive equilibrium can be Nash implemented in an economy with consumption externalities. An economy with consumption externalities is a generalization of the standard public goods economy, and naturally we expect the "free rider problem" to remain. Actually, the incentive problem is much more serious in this economy than in a public goods economy. This is because all prices must be privatized with some tax-subsidy system in order to attain Pareto optimality via the market system in an informationally decentralized manner.

We start in section 2 by laying out the basic framework of emonomies with consumption externalities. In section 3, a "Pigouvian" competitive equilibrium is defined in a pure exchange economy with consumption externalities. The existence of the equilibria and the fundamental theorems of welfare economics, ((i) every Pigouvian competitive equilibrium is Pareto optimal, and (ii) every Pareto optimal allocation is attainable via a Pigouvian competitive equilibrium provided that the initial endowments are suitably redistributed) are proven. As mentioned above, these equilibria have a serious incentive problem when we insist on informational decentralization. This is the issue addressed in section 4 of this paper. We will construct a continuous and feasible mechanism which implements the Pigouvian competitive equilibrium even in the following situation. The mechanism designer does not know either the individuals' preferences or initial endowments. Each individual knows his own preferences and his own initial endowments but not those of others.

2. Environments

Consider a pure exchange economy[1)] with n consumers and $l+1$ commodities. A commodity bundle is denoted by (x, y), where $x \in R_+$ (numéraire without externalities) and $y \in R_+^l$ (social commodities with externalities). The i-th consumer's preference relation is denoted by $\succeq_i$ which is a binary relation[2)] on the set $R_+ \times R_+^l \times R_+^{l(n-1)}$. Let $P_i : R_+ \times R_+^l \times R_+^{l(n-1)} \longrightarrow R_+ \times R_+^l \times R_+^{l(n-1)}$ be the strict upper-contour correspondence. I will assume the following monotonicity assumption of the preferences:

Assumption 2.1: (Monotonicity) For all $(x_i, y_i ; y_{-i}) \in R_+ \times R_+^l \times R_+^{l(n-1)}$ and for all $\varepsilon \in R_{++}^l$

$((x_i, y_i) + \varepsilon; y_{-i}) \in P_i(x_i, y_i ; y_{-i})$.

His initial endowment is given by $(\omega_i^x, \omega_i^y) \in R_+ \times R_+^l$.

Note that I assume implicitly that the "consumption set" is equal to the non-negative orthant, although I do not assume the completeness of the preferences.

The attainable set of this economy is denoted by the following set.

$$A \equiv \Big\{(x, y) \in R_+^n \times R_+^{ln} : \sum_i (x_i - \omega_i^x) = 0$$

and

$$\sum_i (y_i - \omega_i^y) = 0\Big\}.$$

Pareto optimality and individual rationality are defined as follows:

Definition 2.1: $(x^*, y^*) \in A$ *is Pareto optimal* if there is no $(x, y) \in A$ such that for all i,

$$(x_i, y_i, y_{-i}) \in P_i((x_i^*, y_i^* ; y_{-i}^*)).$$

Definition 2.2: $(x, y) \in A$ *is individually rational* if for all i,

$$(\omega_i^x, \omega_i^y ; \omega_{-i}^y) \notin P_i(x_i, y_i ; y_{-i}).$$

3. Competitive Equilibria

3.1. *Definitions*

In this section, we will define a Pigouvian competitive equilibrium. The first definition is a transfer system which takes a role to equate the private marginal cost with the social marginal cost.

Consider two distinct consumers i and j $(i \neq j)$. Let t_{ij} be a vector of transfer rates from i to j. Then if consumer j consumes y_i unit of commodity y, then consumer i pays $t_{ij}(y_j - \omega_j^y)$ for j's consumption. Similarly, consumer j pays $t_{ji}(y_i - \omega_i^y)$ for i's consumption of commodity y of y_i unit. Thus i's net transfer to consumer j is $t_{ij}(y_j - \omega_j^y) - t_{ji}(y_i - \omega_i^y)$. Hence the sum of transfers paid by i is equal to

$$\sum_{j \neq i}\Big(t_{ij}(y_j - w_j^y) - (t_{ji}(y_i - \omega_i^y)\Big)$$

$$= \sum_{j \neq i} t_{ij}(y_j - \omega_j^y) + (-\sum_{j \neq i} t_{ji})(y_i - \omega_i^y).$$

Thus if we write $-\sum_{j \neq i} t_{ji}$ as t_{ii}, which can be interpreted as the tax rate of consumer i, then the total transfer from i can be written as $\sum_j t_{ij}(y_j - \omega_j^y)$.

Formally, the transfer system is defined as follows.

Definition 3.1: $t \in R^{ln^2}$ is called a *transfer system if for all* j,

$$\sum_{i \neq j} t_{ij} = -t_{jj}.$$

The above condition means that the subsidy $(= -t_{jj})$ of the j-th consumer is equal to the sum of the transfers to consumer j.

Remark 3.1: $t \in R^{ln^2}$ is a transfer system if and only if

$$\sum_i \sum_j t_{ij}(y_j - \omega_j^y) = 0 \quad \text{for every } y \in R_+^{ln}.$$

$t_{ij}(y_j - \omega_j^y)$ is an amount of transfer from consumer i to consumer j. Hence this remark means that total transfer is always equal to zero so that the budget constraint of the government is always satisfied, which guarantees the Walras law.

Secondly, we will define the budget set of consumer i in this economy.

Definition 3.2: For each price $p \in R^l$ and a transfer system $t \in R^{ln^2}$, let

$$B_i(p, t) \equiv \Big\{(x_i, y_i ; y_{-i}) \in R_+ \times R_+^{ln} : x_i + py_i \leq \omega_i^x + p\omega_i^y - \sum_j t_{ij}(y_j - \omega_j^y) \text{ and } \sum_j y_j = \sum_j \omega_j^y\Big\}.$$

The second condition in the budget constraint is the balanced condition of social commodities y. Since consumer i specifies his desired consumption level of the others', it is natural for him to consider the balance of demand of commodity y. Note that he must know others' initial endowments in order to know his budget constraint. This creates another informational problem concerning to the following Pigouvian competitive equilibrium.

Definition 3.3 : $(p^*, t^*, x^*, y^*) \in R^l \times R^{ln^2} \times R_+{}^n \times R_+{}^{ln}$ is called a *Pigouvian competitive equilibrium* if

(1) t^* is a transfer system,

(2) $(x_i^*, y_i^*; y_{-i}^*)$ is a maximal element of $\gtrsim_i$ in $B_i(p^*, t^*)$, namely,

(2.1) $(x_i^*, y^*) \in B_i(p^*, t^*)$

(2.2) $P_i(x_i^*, y^*) \cap B_i(p^*, t^*) = 0$

(3) $(x^*, y^*) \in A$.

The corresponding allocation (x^*, y^*) is called *a Pigouvian competitive allocation.*

The meanings of conditions (1) and (3) are clear. Condition (2) means that consumers maximize their preferences given the equilibrium prices and transfers. Namely condition (2) means that the allocation y^* is optimal for each consumer under his own budget set. It should be noted that the Pigouvian equilibrium in this context is different from the equilibrium introduced by Osana (1977) in which y_i^* is optimal given not only prices and transfers but also others' consumption $(y_1^*, \cdots, y_{i-1}^*; y_{i+1}^*, \cdots, y_n^*)$. In fact, our definition of the Pigouvian equilibrium in this context is stronger than that of the equilibrium in Osana (1977), so that we can assure non-wastefulness and individual rationality.

3.2. *Theorems*

Now we can state the following theorems. Note that in most of the following analysis, the convexity of preferences will be assumed. This assumption is a necessary evil, since Calsamiglia (1977) proved the impossibility of realization of Pareto optimal correspondence with finite dimensional message spaces in non-convex environments.

Theorem 3.1 : (Existence) For all i, assume the following :

(1) $P_i(\cdot)$ has an open graph.

(Continuity)

(2) *For all* $(x, y) \in R_+ \times R_+{}^l \times R_+{}^{l(n-1)}$ $(x_i, y) \notin \text{conv}\, P_i(x_i, y)$.[3)]

(Convexity)

(3) $\omega_i^x \in R_{++}$

Then there exists a Pigouvian competitive equilibrium.

Proof : Let us introduce the following notations :

$$X_i \equiv R_+ \times \{(0, \cdots, 0)\} \times R_+{}^{ln} \times \{(0, \cdots, 0)\} \subset R_+ \times R_+{}^{ln(i-1)} \times R_+{}^{ln} \times R_+{}^{ln(n-i)}$$

$$Y \equiv \{(x, y_1, \cdots, y_n) \in R \times R^{ln2} : y_1 = \cdots = y_n, \quad x = 0 \text{ and } \sum_j y_{ij} = 0 \text{ for all } i\}$$

$$\tilde{\omega}_i \equiv (\omega_i^x, 0, \cdots, 0, \omega^y, 0, \cdots, 0) \in X_i.$$

We will extend P_i on X_i in the natural way, that is

$$(x_i, 0, \cdots, 0, y, 0, \cdots, 0) \in P_i(x_i', 0, \cdots, 0, y', 0, \cdots, 0)$$

$$\Longleftrightarrow$$

$$(x_i, y) \in P_i(x', y')$$

Denote

$$\tilde{A} = \left\{(u, v) \in \prod_i X_i \times Y : \sum_i u_i = \sum_i \tilde{\omega}_i + v\right\}.$$

Then $\tilde{A}$ is compact since X_i's are lower bounded and closed. Hence there is a convex and compact set $K \subset R \times R^{ln^2}$ such that proj $x_i\tilde{A} \subset$ int K and proj $y\tilde{A} \subset$ int K. Let $\tilde{X}_i \equiv X_i \cap K$ and $\tilde{Y} \equiv Y \cap K$.

Fix any natural number ν. Consider the following disk as the set of combination of prices and transfers :

$$D^\nu \equiv \{q \in R \times R^{ln^2} : \|q\| \leq \nu\}.$$

For all $q \in D^\nu$, define the budget set of consumer i as

$$C_i(q) \equiv \{u_i \in \tilde{X}_i : (1, q)\, u_i \leq (1, q)\, \tilde{\omega}_i\}.$$

Since $\omega_i^x \in R_{++}$, C_i is a continuous correspondence from D^ν into $\tilde{X}_i$.

Now consider the following $n+2$ players' abstract economy :

First n players : Consumers

Strategy Set : $\tilde{X}_i$

Preference Correspondence : $P_i(\cdot)$

Constraint Correspondence : $C_i(\cdot)$

$(n+1)$-st player : Firm

Strategy Set : $\tilde{Y}$

Preference Correspondence :

$P_f(\cdot)$ which is defined by $\forall q \in D^\nu$ and $\forall_v \in \tilde{Y}$

$P_f(q, v) \equiv \{v' \in \tilde{Y} : (1, q)v' > (1, q)v\}$

Constraint Correspondence : $\tilde{Y}$

$(n+2)$-nd player : Auctioneer

Strategy Set : D^ν

Preference Correspondence : $P_a(\cdot)$ which is defined by

$\forall q \in D^\nu$, $\forall (u_1, \cdots, u_n) \in \prod_{i=1}^{n} \tilde{X}_i$ and $v \in \tilde{Y}$

$$P_a(q, u, v) \equiv \left\{q' \in D^\nu : (1, q')\left(\sum_{i=1}^{n}(u_i - \tilde{\omega}_i) - v\right) > (1, q)\left(\sum_{i=1}^{n}(u_i - \tilde{\omega}_i) - v\right)\right\}$$

Constraint Correspondence : D^ν

Then by Shafer and Sonnenschein (1975), there is a generalized Nash equilibrium $(q^\nu, u^\nu, v^\nu) \in D^\nu \times \prod_{i=1}^{n} \tilde{X}_i \times \tilde{Y}$, which satisfies the following conditions :

$$(1, q^\nu)u_i^\nu \leq (1, q^\nu)\tilde{\omega}_i \tag{1}$$

$$\text{conv } P_i(u^\nu) \cap \{u_i \in \tilde{X}_i : (1, q^\nu)u_i \leq (1, q^\nu)\tilde{\omega}_i\} = 0 \tag{2}$$

$$(1, q^\nu)v^\nu \geq (1, q^\nu)v \quad \forall v \in \tilde{Y} \tag{3}$$

$$(1, q^\nu)\left(\sum_{i=1}^{n}(u_i^\nu - \tilde{\omega}_i) - v^\nu\right) \geq (1, q)\left(\sum_{i=1}^{n}(u_i^\nu - \tilde{\omega}_i) - v^\nu\right) \quad \forall q \in D^\nu \tag{4}$$

Using standard argument one can prove $\|q^\nu\| \not\to \infty$ by the monotonicity of the preferences. Moreover, since $\tilde{X}_i$ and $\tilde{Y}$ are compact, we may assume, without loss of generality, that

$$q^\nu \to q^*, \quad u^\nu \to u^*, \quad \text{and} \quad v^\nu \to v^* \quad \text{as } v \to \infty. \tag{5}$$

Hence using equations (1), (2), (3), and (4), one can assert

$$(1, q^*)u_i^* \leq (1, q^*)\tilde{\omega}_i \tag{6}$$

$$\text{conv } P_i(u^*) \cap \left\{u_i \in \tilde{X}_i : (1, q^*)u_i \leq (1, q^*)\tilde{\omega}_i\right\} = 0 \tag{7}$$

$$(1, q^*)v^* \geq (1, q^*)v \quad \forall v \in \tilde{Y} \tag{8}$$

$$(1, q^*)\left(\sum_{i=1}^{n}(u_i^* - \tilde{\omega}_i) - v^*\right) \geq (1, q)\left(\sum_{i=1}^{n}(u_i^* - \tilde{\omega}_i) - v^*\right) \quad \forall q \in R^{ln^2} \tag{9}$$

Hence if we write : $u_i^* \equiv (x_i^* ; 0, \cdots, 0, y_i^*, 0, \cdots, 0)$ and $v^* \equiv (\bar{x}, \bar{y}, \cdots, \bar{y})$, then using (6) and (9), one can prove the following equations :

$$\sum_{i=1}^{n} x_i^* \leq \bar{x} + \sum_{i}^{n} \omega_i^x$$

$$y_i^* = \bar{y} + \omega^y \equiv y^* \quad \text{for all } i.$$

But since $(\bar{x}, \bar{y}, \cdots, \bar{y}) \in Y$, it follows that $\bar{x}=0$ and $\sum_{i=1}^{n} \bar{y}_i = 0$. Moreover, using (8), one can prove $\sum_{i=1}^{n} q_{ij}^* = \sum_{i=1}^{n} q_{ik}^* \equiv p^*$ for all j and k, and $(1, q^*)v^* = 0$. Hence $(x_1^*, \cdots, x_n^* ; y^*) \in A$.

In order to find the equilibrium transfer system, define : $t_{ij}^* \equiv q_{ij}^*$ if $i \neq j$ and $t_{ii}^* \equiv q_{ii}^* - p^*$. //

Theorem 3. 2 : (Non-Wastefulness) Every Pigouvian competitive allocation is Pareto optimal.

Proof : The proof is straightforward using the standard proof by contradiction. //

Theorem 3. 3 : (Unbiasedness) For all i, assume the following :

(i) $P_i(\cdot)$ is open-valued. (Continuity)

(ii) $P_i(\cdot)$ is convex-valued. (Convexity)

Then every optimal allocation $(x^*, y^*) \in R_{++}^{n} \times R_{++}^{ln}$ can be attained as a Pigouvian competitive allocation provided the initial endowments are suitably redistributed.

Proof : Let

$$D \equiv \Big\{(x\,;\, y_1, \cdots, y_n) \in R \times R^{ln^2} :$$
There exists $(x_1, \cdots, x_n) \in R^n$ such that
$$x = \sum_i x_i \text{ and } (x_i + x_i^*, y_i + y^*) \in P_i(x_i^*, y^*) \ \forall i\Big\}.$$

and

$$F \equiv \Big\{(x\,;\, y_1, \cdots, y_n) \in R \times R^{ln^2} : y_1 = \cdots = y_n,\ x \leq 0, \text{ and } \sum_j y_{ij} = 0 \quad \forall i\Big\}.$$

Then D and F are convex and $D \cap F = 0$ since (x, y^*) is Pareto optimal. Hence there is a hyperplane $(q^x\,;\, q_1^y, \cdots, q_n^y) \in R \times R^{ln^2} \backslash \{0\}$ and $r \in R$ such that

$$q^x x + \sum_i q_i^y y \leq r \quad \forall (x\,;\, y, \cdots, y) \in F \quad (1)$$

$$q^x x + \sum_i q_i^y y_i \geq r \quad \forall (x\,;\, y_1, \cdots, y_n) \in D. \quad (2)$$

By monotonicity, $q^x \geq 0$.

Since $(0\,;\, \cdots, 0) \in F$ and $((\varepsilon, \cdots, \varepsilon)\,;\, 0, \cdots, 0) \in D \ \forall_\varepsilon \in R_{++}$, it follows that $q^x x^* + \sum_i q_i^y y^* = r$. Hence by (1), for all $(x, y) \in R^{ln^2}$ with $x \leq 0$ and $\sum_j y_j = 0$,

$$q^x x^* + \sum_i q_i^y y^* \geq q^x x + \sum_i q_i^y y.$$

So we can show that

$$\sum_i q_{ij}^y = \sum_i q_{ik}^y \equiv p^* \quad \text{for all } j \text{ and } k.$$

Let i be such that $(q^x, q_i^y) \neq 0$. Without loss of generality, assume $i=1$. Then for all $(x_1, y) \in P_1(x_1^*, y^*)$,

$$q^x x_1 + q_1^y y \geq q^x x_1^* + q_1^y y^*$$

Namely

$$P_1(x_1^*, y^*) \cap \{(x_1, y) \in R_+ \times R_+^{ln} : q^x x_1 + q_1^y y < q^x x_1^* + q_1^y y^*\} = 0.$$

Since $P_1(x_1^*, y^*)$ is open,

$$P_1(x_1^*, y^*) \cap \{(x_1, y) \in R_+ \times R_+^{ln} : q^x x_1 + q_1^y y \leq q^x x_1^* + q_1^y y^*\} = 0.$$

Hence by monotonicity, $q^x > 0$. Hence we may assume that

$$q^x = 1 \quad \text{and} \quad \sum_i q_i^y = (\overbrace{P^*, \cdots, P^*}^{n \text{ times}})$$

so that $r=0$. Define

$$t_{ij}^* \equiv q_{ij}^y \quad \text{if } i \neq j$$
$$t_{ii}^* \equiv q_{ii}^y - P^*, \quad \text{and}$$
$$(\omega_i^x, \omega_i^y) \equiv (x_i^*, y_i^*)$$

Then t is a transfer system. Hence one can prove that for all i and for all $(x_i, y) \in P_i(x_i^*, y^*)$,

$$x_i + q_i^y y \geq x_i^* + q_i^y y^*.$$

Namely

$$P_i(x_i^*, y^*) \cap \{(x_i, y) \in R_+ \times R_+^{ln} : x_i + q_i^y y < x_i^* + q_i^y y^*\} = 0.$$

Since $P_i(x_i^*, y^*)$ is open,

$$P_i(x_i^*, y^*) \cap \{(x_i, y) \in R_+ \times R_+^{ln} : x_i + q_i^y y \leq x_i^* + q_i^y y^*\} = 0.$$

Namely

$$P_i(x_i^*, y^*) \cap \Big\{(x_i, y) \in R_+ \times R_+^{ln} : x_i + p^* y_i \leq \omega_i^x + p^* \omega_i^y - \sum_j t_{ij}^* (y_j - \omega_j^y)\Big\} = 0.$$

//

Theorem 3.4 : (Individual Rationality) Every Pigouvian competitive allocation is individually rational.

Proof : Obvious. //

4. Feasible Nash Implementation

This section proves the possibility of feasible Nash implementation of Pigouvian competitive equilibrium when the mechanism designer does not know either the individual preferences or the individual initial endowments. We also assume that each individual only knows his own prefernces and his own initial endowment and does not know the others'. Again, a convexity assumption on preferences plays an important role.

4.1 *Mechanism*

From now on, we will write the i-th consumer's true initial endowments as $\mathring{\omega}_i \equiv (\mathring{\omega}_i^x, \mathring{\omega}_i^y)$. Let us make the following assumptions:

Assumption 4.1: $n \geq 3$.[4)]

Assumption 4.2: $\mathring{\omega}_i \gg 0$.

Assumption 4.3: $\gtrsim_i$ is complete, transitive, and convex.

Assumption 4.4: (Boundary Condition)

$\forall (x_i, y_i) \in \text{int } R_+^{l+1}$,[5)]
$\forall (x_i', y_i') \in \partial R_+^{l+1}$,[6)]
and $\forall y_{-i} \in R_+^{l(n-1)}$,
$(x_i, y_i; y_{-i}) \succ_i (x_i', y_i', y_{-i})$.

Let us consider the following mechanism.

Definition 4.1: (Message Space) For all i, let

$$M^i \equiv R^l \times R^{ln} \times (0, \mathring{\omega}_i] \quad \text{and} \quad M \equiv \prod_i M^i.$$

The representative strategy of consumer i, is denoted by $m_i \equiv (P_i, (y_{ij})_j, \omega_i)$, Which can be interpreted in a following way:

(1) p_i: proposed price.
(2) y_{ij}: proposed total consumption of consumer j.
(3) ω_i: reported initial endowments.

For a given $m \equiv (m_i)_i \equiv (p_i, (y_{ij})_j, \omega_i)_i \in M$, define the following mechanism:

Definition 4.2:

$$\alpha_i(m) \equiv \sum_{k,k' \neq i} (p_k - p_{k'})^2$$

$$\alpha(m) \equiv \sum_i \alpha_i(m)$$

$$\beta_i(m) \equiv \begin{cases} \alpha_i(m)/\alpha(m), & \text{if } \alpha(m) > 0; \\ 1/n & \text{otherwise.} \end{cases}$$

$$p(m) \equiv \sum_i \beta_i(m) p_i$$

Namely, the actual price $p(m)$[7)] is a weighted average of the proposel price p_i by each consumer i with a coefficient $\beta_i(m)$ which is also affected by individuals' strategies. Note that this $p(\cdot)$ is continuous even though $\beta_i(\cdot)$ is discontinuous. The transfer system will be defined in the following way:

Definition 4.3:

$$t_{ij}(m) \equiv y_{i+1,j} - y_{i+2,j}.$$

Note that this $(t_{ij}(m))_{ij}$ is actually a transfer system.

Definition 4.4:

$$D(m) \equiv \Big\{ (y_1, \cdots, y_n) \in R_+^{ln} : \forall i \; p(m) y_i + \sum_j t_{ij}(m)(y_j - \omega_j^y) \leq \omega_i^x + p(m)\omega_i^y \text{ and } \sum_i y_i = \sum_i \omega_i^y \Big\}.$$

Note that the above $D(\cdot)$[8)] is convex-valued and continuous (*i. e.*, both upper semi-and lower semi-continuous.)

Definition 4.5:

$$y_i(m) \equiv \sum_i y_{ij} - \sum_i y_{i,j+1} + \omega_j^y.$$

Definition 4.6:

$$(Y_1, \cdots, Y_n)(m) \equiv \text{argmin}\{\|y - y(m)\| : y \in D(m)\}.$$

and

$$X_i(m) \equiv p(m)\Big(\omega_i^y - Y_i(m)\Big) + \sum_j t_{ij}(m)\Big(\omega_j^y - Y_j(m)\Big) + \omega_i^x.$$

4.2. *Theorems*

Theorem 4.1: This mechanism is continuous and feasible.

Proof: Straightfoward. //

Theorem 4.2: The set of Nash allocations coincides with the set of Pigouvian competitive allocations.

Proof: Let (p^*, t^*, x^*, y^*) be a Pigouvian competitive equilibrium. We will define a strategy profile $m^* = (p_i^*, (y_{ij}^*)_j, \omega_i^*)_j$ in the following way. Let $p_i^* = p^*$ and $\omega_i^* = \mathring{\omega}_i$. In order to define y_{ij}^*, we will conside the following devices: Let $(\tilde{y}_1^*, \cdots, \tilde{y}_n^*)$ be a solution of the following:

$$\tilde{y}_{ih}^* - \tilde{y}_{j+1,h}^* = y_{ih}^* - \mathring{\omega}_{jh}^y$$
$$\forall j = 1, \cdots, n \quad \forall h = 1, \cdots, l.$$

For a fixed $j = 1, \cdots n$ and a fixed $h = 1, \cdots, l$, consider the following linear equation system:

$$\begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ 1 & -1 & 0 & \cdots & 0 \\ & & \cdots & & \\ & & \cdots & & \\ 0 & \cdots & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} y_1^{jh} \\ y_2^{jh} \\ \cdot \\ \cdot \\ y_n^{jh} \end{pmatrix} = \begin{pmatrix} \tilde{y}_{jh}^* \\ t_{njh}^* \\ t_{1jh}^* \\ \cdot \\ t_{n-2,jh}^* \end{pmatrix}$$

According to Walker (1981), the above system of equations has the unique solution (y_i^{jh}) i. Define $y_i^* = (y_i^{jh})_{jh}$. Then it is easy to see the value of the mechanism at m^* coincides with the Pigouvian competitive allocation. Moreover, since for all m_i, $(X_i(m_i, m_{-i}^*), Y(m_i, m_{-i}^*))$ satisfies his budget constraint by the definition of mechanism, one can show

$$(X_i(m^*), Y(m^*)) \gtrsim_i (X_i(m_i, m_{-1}, ^*), Y(m_i, m_{-1}, ^*)).$$

Hence m^* is a Nash equilibrium.

Conversely, let $m^* = (p_i^*, (y_{ij}^*)_j, r_i^*, \omega_i^*)$ be a Nash equilibrium. Suppose that $\omega_i^* \neq \mathring{\omega}_i$. Then by increasing his reported initial endowments, he can attain a larger y_i, which is the contradiction by monotonicity. Hence $\omega_i^* = \mathring{\omega}_i$.

We will prove that for any i, $(X_i(m^*), Y(m^*))$ maximizes his preference relation subject to his budget constraint determined by $p(m^*)$ and $t(m^*)$. It is straightforward to show this $(X_i(m^*), Y(m^*))$ does satisfy his budget constraint. In order to prove the preference maximization, suppose, on the contrary, that there exist i and (x_i, y) such that

$$(x_i, y) \in B_i(p(m^*), t(m^*)) \qquad (1)$$

$$(x_i, y) \succ_i (X_i(m^*), Y(m^*)). \qquad (2)$$

By monotonicity, we can assume, without loss of generality, that the budget constraint is satisfied with equality. By the boundary condition, one can prove that $0 \ll X_k(m^*) \ll \sum_{j=1}^{n} \mathring{\omega}_j^x \quad \forall k$. Hence by the convexity of preferences, taking the convex combination if necessary, we can assume without loss of generality that y is sufficiently close to $Y(m^*)$, so that (x_i, y) is attainable for him, which contradicts the fact that m^* is a Nash equilibrium. //

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Notes

1) For more general case, see Osana (1977).

2) Note that we do not assume either transitivity or completeness of the preferences at this stage.

3) For a given set X, conv X denotes the convex hull of X.

4) When we have only two consumers, some difficulties arise. In particular, one can get some impossibility results concerning Nash implementation of even Walrasian or Lindahl epuilibria. For details, see Kwan and Nakamura (1987).

5) For any set X, int X denotes the topological interior of X.

6) For any set X, ∂X denotes the topological boundary *of* X.

7) This construction of price function is the same as that in Postlewaite and Wettstein (1989).

8) The idea of this definition of the budget correspondence is appeared in Tian (1989).

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