Long Run Equilibrium, Income Distribution among Heterogeneous Classes and Taxation in a Two Sector Growing Economy

Jun Young Kim

I. Introductions

This work examines how taxation has effects on long run equilibrium, capital accumulation and income distribution, extending previous analyses mostly based on a one sector growth model to a two sector growth model with two classes of capitalists and workers. Further this analysis includes redistribution of tax revenue between two classes. In the past works with identical individuals interests are in how taxation affects capital accumulation in the contexts of tax incidence or welfare cost.

Although theoretical model in the fields was developed from Stiglitz and Pasinetti, they are more concerned with the properties of a growth model. They could not combine taxation with a change in the ownership of capital stock derived from redistribution of tax revenue among heterogenous classes. Here we will show this combination can be analyzed on surfaces with 3 dimensions.

The analytic model in this paper is defferent from the past one in the sense that this model uses a two sector growth model with non-identical individuals and considers redistribution between two classes. Hence, our analytic points with this feature of the model are the long run effects of taxation not only on the overall capital labor ratio of the economy, but also on the ownership of capital.

The result in this work indicates that changing a consumption tax rate would be neutral with respect to the capital accumulation or increase it under certain conditions. So the results appeared in Theorem 1 and 2 are different from them derived by Barro, Feldstein and Summers. Because their results are based on switching from an income tax to a consumption tax or a wage tax. But results in this paper are made by analyzing the effect of a consumption tax on capital accumulation from the view point of redistribution between two classes, which is not the central point of the previous literature. It is policy implications of the results in this paper that increases in a consumption tax rate would stimulate capital accumulation or be neutral to it with redistribution of the ownership of capital, depending on the elasticities of saving of two classes, the relative factor intensities between two sectors and a fraction of redistribution of tax revenue to each class.

II. Theoretical Model

Assume that the individual's taste for present versus future consumption is related to the source of its income. There are two sources of income in the model: an individual may provide capital or labor (or possibly both), where he is capitalist or worker.

Assume that the capitalists who get their incomes from capital have a greater maginal propensity to save than the workers who get their income from both wages and return to saving. In this model, workers are assumed to own a portion of the capital stock of the economy. In the long run this portion is not given, but is endogenous variable.

1. Supply side of the model

* I appreciate for referee's comments on my paper, which gave me new insight to making a revision of my initial paper. I retain responsibility for all remaining errors.

Consider a two sector (a consumption goods sector and an investment goods sector) model. Let $C=F(K_C, L_C)$ be the output of consumption goods as a function of quantities of capital (K_C) and labor (L_C) employed in their production. Let $I=G(K_I, L_I)$ be the output of investment goods as a function of quantities of capital (K_I) and labor (L_I) employed in their production.

(A. 1) Assume that capital does not depreciate, and no technical change occurs.

(A. 2) Assume that the production functions are homogeneous of degree one and strictly quasi-concave.

(A. 3) Both factors are fully employed, and their markets are perfectly competitive.

Then output perhead can be written as

$$(2.1)\quad x_C=C/L=(L_C/L)f(k_C) = \frac{k-k_I}{k_C-k_I}f(k_C)=x_C(k, p)$$

$$(2.2)\quad x_I=I/L=(L_I/L)g(k_I) = \frac{k_C-k}{k_C-k_I}g(k_I)=x_I(k, p)$$

where $k=K/L$, $k_C=K_C/L$, $k_I=K_I/L$ and p is the price of capital goods in terms of consumption goods, which are taken as numeraire. Labor forces increase at the rate of n.

The last step in (2. 1) and (2. 2) follows directly from the fact that k_C and k_I are a function of p only from the following profit maximizing conditions:

$$(2.3)\quad f'(k_C)=pg'(k_I)=r$$

$$(2.4)\quad f(k_C)-k_Cf'(k_C) = p[g(k_I)-k_Ig'(k_I)] = w$$

where a prime on a functions denotes its derivative, r is the rental price of capital and w is the wage rate.

2. Demand side of the model

Let Y_l be the part of per capita national income that accrues to workers, and Y_k be the capitalists' part, i. e.,

$$Y_l = w+rk_l \quad \text{where } k_l = K_l/L$$
$$Y_k = rk_k \quad \text{where } k_k = K_k/L$$

and where K_l is the quantity of capital owned by workers, and K_k is the quantity of capital owned by capitalists ($K_l+K_k=K$). Clearly,

$$Y = Y_l+Y_k = w+r(k_l+k_k) = w+rk.$$

If all savings are invested in a soiciety, the functional form of the demand for investment good is

$$(2.5)\quad pd_I = S_k(R)rk_k+S_l(R)(w+rk_l) = S_lY+(S_k-S_l)rk_k = d_I'(k, p, k_k)$$

$$0 < S_l < S_k < 1,\ S_k' \geqslant 0,\ S_l' \geqslant 0$$

where S_k and S_l are the marginal propensity to save of capitalists and workers, respectively, and $R=r/p$.

Similarly, the functional form of the demand for consumption good is

$$(2.6)\quad d_C = (1-S_k(R))rk_k + (1'-S_l(R))(w+rk_k) = d_C'(k, p, k_k).$$

3. Equilibrium, its existence and uniqueness

(1) Short run equilibrium

Define the excess supply function of investment good and cnsumption good, respectively as follows:

$$F_I(k, p, k_k) = px_I(k, p)-d_I'(k, p, k_k)$$
$$E_C(k, p, k_k) = x_C(k, p-d_C'(k, p, k_k)$$

Define the short run equilibrium by (k, p, k_k) such that $E_I=0$ and $E_C=0$.

From the income cnstraints

$$x_C(k, p)+px_I(k, p) = d_C'(k, p, k_k)+d_I'(k, p, k_k)$$

or $$x_C(k, p)-d_C'(k, p, k_k) = d_I'(k, p, k_k)-px_I(k, p)$$

So $$E_C(k, p, k_k)=-E_I(k, p, k_k).$$

Therefore, whenever $E_C(k, p, k_k)=0$, it must be true that $E_I(k, p, k_k)=0$, too (Walras Law). For convenience to ananysis of the short run equilibrium, we choose the excess supply function of investment goods.

Let's see the properties of the short run equilibrium by deriving a surface with 3 dimensions (k, p, k_k) for which the market is in a short run equilibrium. By differentiating $E_I(k, p, k_k)=0$ with respect to k, p and k_k (for simplicity take S_k and S_l are constant, independent of R), we can derive the slopes of the surface of $E_I(k, p, k_k)=0$ in its different directions.

(2.7) $dk/dk_k|E_I=0, p=\text{constant}$
$$=(S_k-S_l)r/\left(p\frac{dx_I}{dk}-S_l\frac{dY}{dk}\right)$$

(2.8) $dp/dk|E_I=0, k_k=\text{constant}$
$$=-\frac{p(dx_I/dk)-S_l(dY/dk)}{p(dk_I/dp)+x_I(1-S_l)-(S_k-S_l)k_k(dY/dp)}$$

(2.9) $dp/dk_k|E_I=0, k=\text{constant}$
$$=\frac{r(S_k-S_l)}{p(dx_I/dp)+x_I(1-S_l)-(S_k-S_l)k_k(dY/dp)}$$

If the production of consumption goods is capital intensive, the sign of (2.7) is negative, and the sign of (2.8) and (2.9) are positive. The sign of these derivatives make a sense in the market equilibrium condition of (2.5.). Under the assumption that the production of investment goods is relatively labor intensive, an increase in the quantity of capital owned by capitalists would decrease the production of the goods, and thereby reduce capital labor ratio. On the other hand, an increase in capital labor ratio or an increase in the quantity of capital owned by capitalists results in excess demand of the goods in equation (2.5), and hence the price would increase for short run equilibrium.

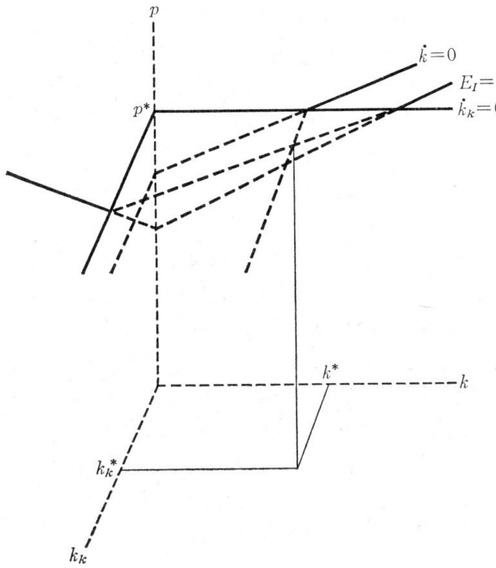
If the production of investment goods is capital intensive the slopes of the surface $E_I(k, p, k_k)=0$ in its different directions would be ambiguous, depending on the relative sizes of $p(dx_I/dk)$ versus $S_l(dY/dk)$, and $p(dx_I/dp)+x_I(1-S_l)$ versus $(S_k-S_l)k_k(dr/dp)$. For the case where the production of consumption goods is relatively capital intensive, the surface $E_I(k, p, k_k)=0$ is drown in Figure 1.

(2) Long run equilibrium

The dynamic system of capital accumulation in the model is $\dot{k}=x_I(k, p)-nk$ and $\dot{k}_k=S_k r k_k-nk_k$. Define the long run equilibrium as (k, p, k_k) such that (k, p, k_k) clears markets, and makes k and k_k stationary over time (t):

[Figure 1]

The long run equilibriun (k^*, p^*, k_k^*) $(k_C>k_I)$



(2.10) $E_I(k, p, k_k)=0$
$$dk/dt=\dot{k}=0 \text{ and } dk_k/dt=d\dot{k}_k=0.$$

So let's derive two more surfaces of $\dot{k}=0$ and $\dot{k}_k=0$. The $\dot{k}=0$ surface's equation is $x_I(k, p)=nk$ which is independent of k_k. If the consumption goods are relatively capital intensive, the $\dot{k}=0$ surface has the shape as in Figure 1 from (2.8) and (2.9). The $\dot{k}_k=0$ surface's equation is

(2.11) $S_k r k_k=nk_k$ or $r=n/S_k$

which implies the price that makes r equal to n/S_k keeps $\dot{k}_k=0$, regardless of k and k_k. Then the surface $\dot{k}_k=0$ is depicted as in Figure 1. Diagramatically the long run equilibrium (k^*, p^*, k_k^*) can be found, where the above three surfaces are intersected as in Figure 1, when the consumption goods are relatively capital intensive.

As we already discussed, when the production of investment goods is however relatively capital intensive, the slopes of $\dot{k}=0$ and $E_I=0$ are ambiguous in the equations (2.7) through (2.9). But, if $p(dx_I/dk)<S_l(dY/dk)$ and $p(dx_I/dp)+x_I(1-S_l)<(S_k-S_l)k_k(dr/dp)$, then for $k_C>k_I$

$$dk/dk_k|E_I=0, p=\text{constant}<0 \text{ in } (2.7)$$

$dp/dk|E_I = 0, k_k = \text{constant} < 0$ in (2. 8)

$dp/dk_k|E_I = 0, k = \text{constant} < 0$ in (2. 9)

In this case, the change in k_k requires k and p to move in the opposite directions at the equilibrium : for an increase (decrease) in k_k, k is needed to be increased (decreased) and p to be decreased (increased).

(3) Existence and uniqueness of equilibrium

From (2. 10) r is uniquely determined. Hence from the marginal productivity equal factor price relations in the supply model, k_C and k_I are uniquely determined, where the production functions of both sectors are strictly quasi-concave by (A. 2). So are k and k_k. By $E_I=0$, so is p. But from the condition of the long run equilibrium (2. 10), $\dot{k}=0$ and $\dot{k}_k=0$ imply that $\dot{k}_l=0$. The surface $\dot{k}_l=0$ has the equation as :

(2. 12) $S_l Y = S_l r(k-k_l) + nk_l$
$= nk - nk_C + S_l r k_C$

or $S_l Y = nk + k_C(S_l r - n)$.

Substitute (2. 11) into (2. 12). Then

(2. 13) $S_l Y = nk + k_C(S_l/S_k - 1)n$.

By the assumption of $S_l < S_k$, $S_l Y < nk$ with a positive value of k_C.

Hence, the long run equilibrium values $(k^*, p^*, \dot{k}_k)$ exist uniquely if and only if $S_l Y^* < nk^* = x_I(k^*, p^*)$, or equivalently $S_l/S_k < r^* k^*/Y^*$.

This condition states that the ratio of the savings propensity of workers to that of capitalists must be less than the share of capital. So three surfaces of $\dot{k}=0$, $E_I=0$ and $\dot{k}_k=0$ have lower and upper boundaries for $S_l/S_k/r/Y < k < Y/n$ from $x_I = k$ and $x_I < Y$.

III. Long Run Equilibrium and Distributional Effect under Tax on the Output of One Sector : $S_k' = S_l' = 0$

On the basis of the previous theoretical analysis, we will discuss the effect of tax (t) imposed on the output of one sector on the long run equilibrium and income distribution by assuming that the marginal propensities to save are independent of the rate of return to investment ($S_k' = S_l' = 0$). Hence, on the demand side of the model, we will include the redistribution effect, and on the supply side of the model the substitution effect. In the next section, the assumption of $S_k' = S_l' = 0$ will be relaxed.

The tax can be imposed on either consumption goods or investment goods, changing the relative price $p/(1-t)$ or $p(1-t)$, respectively. Then in equations (2. 1) through (2. 4) p is replaced by $p/(1-t)$ or $p(1-t)$ according to the tax on consumption goods or investment goods. Hence, in functional form equation of (2. 1) and (2. 2) under taxation are rewritten as $x_C = x_C(k, p, t)$ and $x_I = x_I(k, p, t)$ and from equations of (2. 3) and (2. 4) r and w are the function of p and t, i. e., $r = r(p, t)$ and $w = w(p, t)$. Since $Y = w(p, t) + r(p, t)k$, $Y = Y(k, p, t)$ in the functional form.

If all savings are invested in a society, the functional form of the demand for investment good is

(3. 1) $d_I' = S_l Y(k, p, t)$
$+ (S_k - S_l)[r(p, t)k_k + \theta T]$

where T is the per capita proceeds of the tax, and θ $(0 \leqslant \theta \leqslant 1)$ is the fraction of them redistributed to capitalists.

For the market equilibrium

(3. 2) $px_I(k, p, t) = S_l Y(k, p, t)$
$+ (S_k - S_l)[r(p, t)k_k + \theta T]$
$= d_I'(k, p, k_k, t)$

The excess supply function is

(3. 3) $E_I(k, p, t)$
$= px_I(k, p, t) - d_I'(k, p, k_k, t)$

which must be zero for the short run equilibrium.

The $\dot{k}=0$ surface's equation is $x_I(k, p, t) = nk$.

The $\dot{k}_k=0$ surface's equation is

(3. 4) $S_k(rk_k + \theta T) = nk_k$.

Let's analyze the effect of taxation on each surface.

The effect of taxation on the short run equilibrium is :

(3. 5) $\Psi_{pt}|E_I=0$

$$=\begin{cases} -\dfrac{(t/1-t)A-(S_k-S_l)(1+(t/1-t)\eta_{xc\bar{p}})\theta T}{x_I\bar{p}+A-(S_k-S_l)\eta_{xc\bar{p}}\theta T} & \text{if the tax is imposed on consumption goods,} \\ \dfrac{(t/1-t)A+(S_k-S_l)(1-(t/1-t)\eta_{x_I\bar{p}})\theta T}{x_I\bar{p}+A-(S_k-S_l)(1+\eta_{x_I\bar{p}})\theta T} & \text{if the tax is imposed on investment goods,} \end{cases}$$

where $A=x_I\bar{p}\eta_{x_I\bar{p}}-S_l x_I\bar{p}-(S_k-S_l)e_{r_p}-rk_k$

and $\eta_{x_I\bar{p}}=(\partial x_I/\partial\bar{p})(\bar{p}/x_I)$,
$\eta_{xc\bar{p}}=(\partial x_C/\partial\bar{p})(\bar{p}/x_C)$,
$\Psi_{pt}=(\partial p/\partial t)(t/p)$,
$e_{r\bar{p}}=(\partial r/\partial\bar{p})(\bar{p}/r)$, $\bar{p}=p/1-t$

for the tax on consumption goods, and $\bar{p}=p(1-t)$ for the tax on investment goods.

The equation (3. 5) tells how the price changes with respect to a change in the tax rate (in the elasticity form) in order to maintain the short run equilibrium on $E_I=0$ surface where $S_k'=S_l'=0$.

The effect of taxation on the surface of $\dot{k}=0$ is:

(3. 6) $\Psi_{pt}|\dot{k}=0$

$$=\begin{cases} -(t/1-t) & \text{if the tax is imposed on consumption goods.} \\ (t/1-t) & \text{if the tax is imposed on investment goods.} \end{cases}$$

The effect of taxation on the surface of $\dot{k}_k=0$ is:

(3. 7) $\Psi_{pt}|\dot{k}_k=0$

$$=\begin{cases} -\dfrac{(t/1-t)(e_{r\bar{p}}rk_k+\theta T\eta_{xc\bar{p}})+\theta T}{e_{r\bar{p}}rk_k+\theta T\eta_{xc\bar{p}}} & \text{if the tax is imposed on consumption goods,} \\ \dfrac{(t/1-t)(e_{r\bar{p}}rk_k+\theta T\eta_{x_I\bar{p}})-\theta T}{e_{r\bar{p}}rk_k+\theta T\eta_{x_I\bar{p}}+\theta T} & \text{if the tax is imposed on investment goods,} \end{cases}$$

where $\bar{p}=p/1-t$ for the tax on consumption goods and $\bar{p}=p(1-t)$ for the tax on investment goods.

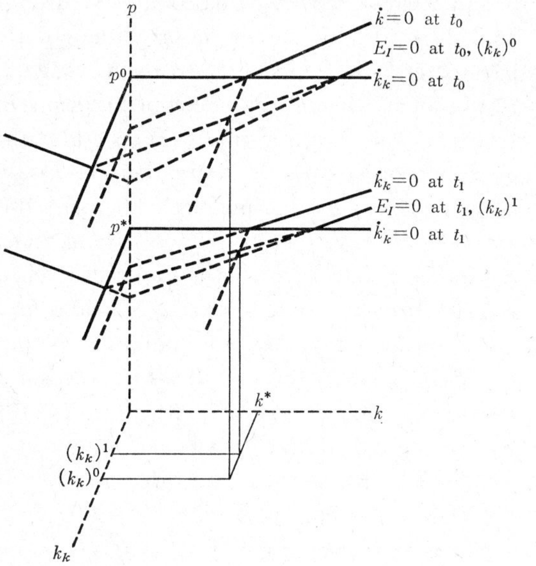
By assuming $S_k=S_l$ the equation of (3. 5) is reduced to the model with identical individuals. The effect of taxation on the long run equilibrium represented by equations (3. 5) through (3. 7) depends on the empirical values of the parameters of the model, i. e., $e_{r\bar{p}}$, S_k, S_l, $\eta_{x_I\bar{p}}$, $\eta_{xc\bar{p}}$, and θ. Hence, the effect of taxation on the long run equilibrium is in general embiguous. In order to have more definite results on tax policy implications, let's assume a tax on consumption goods, which is relatively capital intensive, with the proceeds being redistributed to workers, i. e., $k_C>k_I$ and $\theta=0$. Then from equations (3. 5) through (3. 7) $\Psi_{pt}|\dot{k}=0=\Psi_{pt}|\dot{k}_k=0=-(t/1-t)$. That is, both surfaces of $\dot{k}=0$ and $\dot{k}_k=0$ shift in the same direction by the same distance. From equation (3. 5) $E_I=0$ locus shift in the same direction, but by less than the other two loci.

Since $-\Psi_{pt}|\dot{k}=0=-\Psi_{pt}|\dot{k}_k=0>-\Psi_{pt}|E_I=0$ and the surface of $\dot{k}_k$ shifts down by increasing a consumption tax, the long run equilibrium requires the equilibrium price to be decreased to p^*, having investment goods in excess demands under fixed k and k_k from (3. 3). But a fall in k_k will increase the fall in p needed to clear the markets, without altering the necessity of the fall in p needed to keep $\dot{k}=\dot{k}_k=0$. Therefore a fall in k_k, in fact, makes compatible the fall in p needed to satisfy the long run equilibrium conditions: $\dot{k}=\dot{k}_k=E_I=0$. But the total capital-labor ratio is not affected by changing a consumption tax rate, since changing a cnsumption tax does not generate substitution effects on demand side in which $S_k'=S_l'=0$, and the decrease in k_k is just enough to eliminate an excess demands for investment goods raised by the fall in price.

Hence, the tax on a consumption goods, assumed to be relatively capital intensive with $\theta=0$, is neutral in the sense that the capital-labor ratio doesn't change, only shifting the ownership of capital toward workers, away from capitalists. This can be shown on the plane (k, k_k, p) as in Figure 2, and so we have proved the following Theorem.

[Figure 2]

The effect of an increase in a consumption tax on long run equilibrium ($k_C > k_I, \theta=0, t_1 > t_0, (k_k)^1 > (k_k)^0$)



Theorem 1

Assume $k_C > k_I$, $S_k' = S_l' = 0$, and $\theta = 0$. Then the increase in a consumption tax will be neutral to the total capital labor ratio, but redistribute capital from the capitalists to the wokers.

This Theorem shows by introducing the redistribution effect among heterogenous classes traditional analyses on the effects of taxation upon k should be reexamined in the policy implications of taxation. On a tax policy side this result suggests a consumption tax to be redistributed to workers has strong distributional effect between the capitalist and the worker with the capital-labor ratio in a society unaffected.

Equation (3. 5), (3. 6) and (3. 7) provide a frame-work to analyze the different cases that may be constructed by assuming different values of θ and making assumptions on which good is taxed and on factor intensities.

IV. Long Run Equilibrium and Distributional Effect under Tax on the Output of One Sector : $S_k' > 0$, $S_l' > 0$

This section is a natural extension of the previous analysis in the sense that we now allow the elasticity of savings ($S'(R)$) to be positive. This modifies the interpretation of (3. 3) and (3. 4) in which S_k and S_l are no longer constant, but increasing functions of R. This extension gives theoretically important meaning in that from taxation on the demand side we introduce both redistribution effect and substitution effect, and on the supply side substitution effect. The $\dot{k}=0$ locus is not affected by this extension, since it does not depend on S_k and S_l, while the $E_I=0$ locus and the $\dot{k}_k=0$ locus are functions of S_k and S_l.

From the equation (3. 3) we get the effect of taxation on the short run equilibrium. Notice that if we make $S_k' = S_l' = 0$ in (4. 1), we get equation (3. 5).

(4. 1) $\Psi_{pt} | E_I = 0$

$$= \begin{cases} -\dfrac{(t/1-t)(A-R(B+(S'_k-S_l')\theta T)e_{r\bar{p}}) - (S_k-S_l)(1+(t/1-t)\eta_{xc\bar{p}})\theta T}{x_I\bar{p}+A-R(B+(S_k'-S_l')\theta T)(e_{r\bar{p}}-1)-(S_k-S_l)_{xc\bar{p}}\eta\theta T} & \text{if the tax is imposd on consumption goods,} \\ \dfrac{(t/1-t)(A-R(B+(S_k'-S_l')\theta T)e_{r\bar{p}}) + (S_k-S_l)(1-(t/1-t)\eta_{x_I\bar{p}})\theta T}{x_I\bar{p}+A-R(B+(S_k'-S_l')\theta T)(e_{r\bar{p}}-1)-(S_k-S_l)(1+\eta_{x_I\bar{p}})\theta T} & \text{if the tax is imposed on investment goods,} \end{cases}$$

where $\bar{p} = p/1-t$ for a consumption tax and $p = p(1-t)$ for an investment tax, $B = S_l' Y + (S_k' - S_l') r k_k$, $R(B + (S_k' - S_l')\theta T)e_{r\bar{p}}$ which reflects the substitution effect on the demand for investment goods resulted from the change in the marginal propensity to saving by taxation. The equation (4. 1) shows how the price changes with respect to a change in the tax rate (in the elasticity form) on $E_I=0$ surface, where $S_k' > 0$ and $S_l' > 0$. By making $S_k' = S_l' = 0$, $(B + (S_k' - S_l')\theta T e_{r\bar{p}}) = 0$ and we get equation (3. 5) again.

From the equation (3. 4) we get the effect of taxation on $\dot{k}_k = 0$ surface :

(4. 2) $\Psi_{pt}|\dot{k}_k = 0$

$$= \begin{cases} -\dfrac{(/1-t)\,C+\theta T}{C-\varepsilon_{S_kR}(rk_k+\theta T)} & \text{if the tax is imposed on consumption goods,} \\ \dfrac{(t/1-1)\,C-\theta T}{C-\varepsilon_{S_kR}(rk_k+\theta T)} & \text{if the tax is imposed on investment goods,} \end{cases}$$

where $C=e_{r\bar{p}}rk_k+\theta T\eta_{xc\bar{p}}+\varepsilon_{S_kR}e_{r\bar{p}}(rk_k+\theta T)$ and $\varepsilon_{S_kR}=(\partial S_k/\partial R)(R/S_k)$, $\bar{p}=p/1-t$ for a consumption tax, and $\bar{p}=p(1-t)$ for an investment tax.

The equation (4. 2) shows the change in the price resulted from a change in the tax rate (in the elasticity form) on $\dot{k}_k=0$ surface.

Again, by making $\varepsilon_{S_kR}=0$ in (4. 2), we get equation (3. 7). Equations (4. 1) and (4. 2) tell that the effect of taxation on the long run equilibrium also depends on factor intensities, $e_{r\bar{p}}$, $\eta_{xc\bar{p}}$, $\eta_{xI\bar{p}}$, ε_{S_kR} and θ. Hence, as in the previous section, let's assume a tax on consumption goods, which are relatively capital intensive, with the proceeds being redistributed to workers, i. e., $k_C>k_I$ and $\theta=0$. Then from the equation (3. 6) in order to keep $\dot{k}=0$ we need a fall in price as indicated by $\Psi_{pt}|\dot{k}=0=-(t/1-t)$.

Equation (4. 1) reduces, for $\theta=0$, to

(4. 3) $$\Psi_{pt}|E_I=0=-\frac{(t/1-t)(A-RBe_{r\bar{p}})}{x_I\bar{p}+A-RB(e_{r\bar{p}}-1)}$$

which is smaller than $(t/1-t)$ in absolute value.

And equation (4. 2) reduces, for $\theta=0$, to

(4. 4) $$\Psi pt|\dot{k}_k=0=-\frac{t/1-t}{1-\dfrac{ES_kR}{e_{r\bar{p}}(\varepsilon_{S_kR}+1)}}$$

which is smaller than $(t/1-t)$ in absolute value because $e_{r\bar{p}}<0$ for $k_C>k_I$ and $\varepsilon_{S_kR}>0$.

Therefore, we have

(4. 5) $-\Psi pt|E_I=0<-\Psi pt|\dot{k}=0$

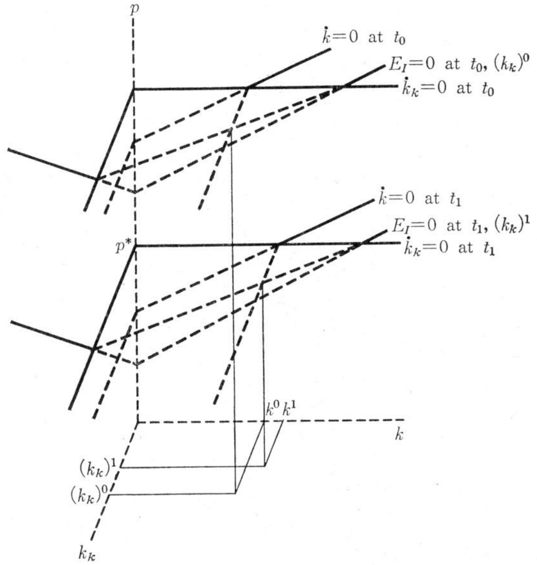
and $-\Psi pt|\dot{k}_k=0<-\Psi pt|\dot{k}=0$.

That is, the market clearing process will provide at constant k and k_k, a fall in price that is smaller than the fall needed for either k_k or k to be constant over time. Therefore, we expect the long run equilibrium to bring out changes in k and/or k_k.

[Figure 3]

The effect of an increase in a consumption tax on the long run equilibrium : Case 1

($k_C>k_I$, $\theta=0$, $t_1>t_0$, $(k_k)^1<(k_k)^0$)



In order to see the effect of a consumption good tax on k and k_k, we have to know the ordering among ${}^{\Psi}pt|E_I=0$, ${}^{\Psi}pt|\dot{k}=0$ and ${}^{\Psi}pt|\dot{k}_k=0$. But the ordering between ${}^{\Psi}pt|E_I=0$ and ${}^{\Psi}pt|\dot{k}_k=0$ is ambiguous. Hence we will consider both cases of the ordering among them :

Case 1 :

$$-{}^{\Psi}pt|E_I=0<-{}^{\Psi}pt|\dot{k}_k=0<-{}^{\Psi}pt|\dot{k}=0$$

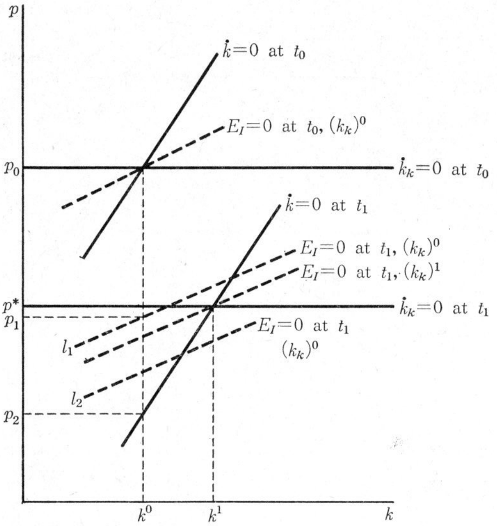
and

Case 2:

$$-{}^{\Psi}pt|\dot{k}_k=0<-{}^{\Psi}pt|E_I=0<-{}^{\Psi}pt|\dot{k}=0.$$

Let's consider Case 1. Since the surface of $\dot{k}=0$ shifts down, the long run equilibrium requires the equilibrium price to fall. At (k^0, $k_k{}^0$, p^*) in Figure 3 there exist excess damands for investment goods and $x_I<nk$. By the way a fall in k_k increases in p needed to clear the markets, thus it makes the fall in p needed to have $E_I=\dot{k}=\dot{k}_k=0$ closer to each other. However, a fall in k_k is not able to affect the fall in p peeded to keep $\dot{k}=0$. That is a fall in k_k alone is not enough to provide the long run equilibrium. So we must also have an increase

[Figure 4]
The effect of an increase in a consumption tax on the long run equilibrium : Case 2
($k_C > k_I, \theta=0, t_1 > t_0, k^1 > k^0$)



in k, which reduces the necessity of the fall in p needed to keep $\dot{k}=0$ by more than the necessity of the fall in p needed to clear the markets. Then our result for this case is that k will rise, k_k will fall, and thereby k_l increases. It is, therefore, clear that k_l will rise by more in this case than in the case previously discussed in which $S_k'=S_l'=0$.

Next, let's consider Case 2. We can also analyze the case in Figure 4 conveniently on (k, p) plane. In this case, an increase in a consumption tax rate shifts $E_I=0$ schedule downward to l_1 or l_2. Whether it is l_1 or l_2, the change in k_k alone cannot have the long run equilibrium. Hence k should also increase to k^1 for the long run equilibrium. But in this case, it is not clear whether k_k would increase or decrease by taxation, but an increase in the consumption tax raises the total capital-labor ratio.

Therefore, our results for both cases indicate that k will rise by increasing a consumption tax. So we have proved the following Theorem.

Theorem 2

Assume $k_C > k_I$, $S_k' > 0$, $S_l' > 0$ and $\theta=0$. Then the increase in a consumption tax will raise the total capital-labor ratio.

This Theorem shows the increase in a consumption tax would raise k or income under certain conditions, when we introduce distrubutional effect among classes, which was neglected by previous literatures.

V. Conclusion

In this work we have analyzed the dynamic tax effects on long run equilibrium and income distribution in a two sector growth model with non-identical individuals of capitalists and workers. We extend the previous analyses to introducing distributional effect among classes and to how taxation affects the long run equilibrium on (p, k, k_k) surface. As we obtained Theorem on a concumption tax, the neutrality to the total capital-labor ratio by changing a consumption tax rate does not hold in general, except for which we impose some restrictions on saving behavior, tax scheme and the relative factor intensities between two sectors.

However, according to Theorem 1 under certain conditions, we will redistribute capital or income from the capitalist to the worker through changing the tax rate on consumption goods with income or capital accumulation unchanged. Theorem 2 implies that an increase in a consumption tax rate will even increase capital accumulation under certain conditions, but to whom capital or income is distributed is ambiguous. Hence, by introducing distributional effect among heterogenous classes our results suggest different insight on a tax policy from other ones, showing that the effects of taxation on the long run equilibrium and income distribution have to be reexamined by equation (3. 5) to (3. 7) and (4. 1) to (4. 2).

(received July 24, 1987, accepted August 22, 1988. Dept. of Economics, Sung Kyun Kwan University)

References

[1] Feldstein, M. A., "Incidence of a Capital In-

come Tax in a Growing Economy with Variable Saving Rate," *Review of Economic Studies*, 1974, pp. 505–513.

[2] ——, "Tax Incidence of a Growing Economy with Variable Factor Supply," *Quarterly Journal of Economics*, 1974, pp. 551–573.

[3] Ezawa, T., "In the Uniqueness and Stability Condition of a Balanced Growth Path in Neoclassical Two Sector Models," *Journal of Economic Theory II*, 1970, pp. 427–436.

[4] Pasinetti, L. L., "Rate of Profit and Income Distribution in Relation to the Rate Economic Growth," *Review of Economic Studies*, 1962, pp. 267–278.

[5] Solow, R., "Note on Uzawa's Two-Sector Model of Economic Growth," *Reviw of Economic Studies*, 1961 –1962, pp. 48–50.

[6] Stiglitz, J. E., "A Two Sector Two-Class Model of Economic Growth," *Review of Economic Studies*, 1967, pp. 227–238.

[7] Summers, L. H., "Capital Taxation and Accumulation in a Life Cycle Growth Model," *The American Economic Review*, Vol. 71, No. 4, pp. 533–544.

[8] Uzawa H., "Optimal Growth in a Two-Sector Model of Capital Accumulation," *Review of Economic Studies*, 1964, pp. 1–24.