Some New Approaches to the Solution of the Problem of Foreign Trade Optimization

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1. The Problem

Trading nations have always tried to gain from selling goods with their own production costs lower than those of their competitors and buying goods which are comparatively more expensive to produce at home than abroad. This practice was termed by D. Ricardo "the principle of comparative costs" [1].

However the full application of the principle encounters a serious difficulty. The solution to the problem of maximum gain from trade is not an easy one to obtain when the number of products and countries is large. Both the classical theory of Ricardo and the Hecksher–Ohlin theory [2, 3] usually consider examples using only two or three goods, one or two production factors, and two trading countries [4–10].

This difficulty has led some writers to model foreign trade as an optimization problem. Models of this type cannot overcome the difficulty wholly because of the limited capabilities of solving largescale problems by means of modern computers. Consequently, the attempts to apply foreign trade optimization models have been, in most cases, based on the use of some aggregated commodity groups. It should be noted that the validity of such aggregation in terms of the necessary conditions that must be met has not been proved. The number of "aggregated goods" was usually limited to several tens [11–15].

It is worth noting that local (branch, sector, commodity group) models can be effectively applied if the goods are not aggregated or aggregated under the classical aggregation theory conditions. But if analysis is confined to a comparatively small local model, then, sooner or later, one faces the problem of the coordination of the model's solution with other such local solutions. Thus, in this case, the difficulty of solving large-scale problems simply takes the form of coordinating local solutions.

A well-known theoretical approach toward solving large-scale programming problems without any information loss rests on the use of the so-called decompositional and iterative methods. These reduce large problems to solving a number of local problems with fewer dimensions. These methods are capable of solving problems of significantly greater dimension than one could solve earlier; in fact containing up to several thousand variables. The possibility of applying these methods is reviewed in [16–20]. The difficulties connected with large dimension can also be solved by iterative coordination of calculations of an aggregated model with the solutions of some local problems in which a detailed nomenclature of products is used. This approach is known as iterative aggregation [21, 22]. In this paper we apply it for the first time to obtain a solution of a foreign trade optimization model.

The methods of iterative aggregation have certain advantages compared with the decomposition and other iterative methods. In particular, in many experimental and in some practical calculations of problems of moderate dimension these methods showed quick convergence to the solution. These methods have a rather natural economic interpretation because they can be said to represent the process of flexible management of activities of independent local units by the center. Thus they can be interpreted in terms of flexible planning or short-term management of plan fulfillment. They can also be interpreted as schemes of state regulation of the work of private firms [23].

In this paper we construct an iterative aggregation scheme, taking into account the specific features of the problem of production and foreign trade optimization. The algorithm is not merely some modification of the iterative aggregation method. It offers a possible formula for the construction of a management system and the joint optimization of production and foreign trade. This is the reason it is being considered here.

The model and the algorithm are described in reference to the national economy as a whole. However, in practice they can be applied for some small and medium industries only, in which case the production and foreign trade targets of other industries would be assumed as given. This means that the suggested method can be used, in the first instance, for the coordination of parts of the national economy and possibly only later for the whole economy.

2. A Basic Model of Production and Foreign Trade Optimization

The model of production and foreign trade we shall consider has the following form.

1. Product demand-supply constraints:

$$\sum_{j}\sum_{k\in K_j} a_{gk}x_k+f_g+y_g+\sum_{d\in D\cup D'} c_{gd}\leqslant\sum_{d\in D\cup D'}\hat{c}_{gd},\qquad g=1,\cdots,G. \tag{2.1}$$

2. Resource constraints:

$$\sum_{j}\sum_{k\in K_j} b_{sk}x_k\leqslant B_s. \tag{2.2}$$

3. Constraints on trade balances in markets with nonconvertible currencies:

$$\sum_{g}[w_{gd}(c_{gd})-\hat{w}_{gd}(\hat{c}_{gd})]\geqslant\Delta_d,\ d\in D. \tag{2.3}$$

4. Constraints on the balance of trade in all markets with convertible currencies:

$$\sum_{d\in D'}\xi_d\sum_{g}[w_{gd}(c_{gd})-\hat{w}_{gd}(\hat{c}_{gd})]\geqslant\Delta. \tag{2.4}$$

5. Nonnegativeness of variables:

$$x_k\geqslant 0,\quad y_g\geqslant 0,\quad c_{gd}\geqslant 0,\quad \hat{c}_{gd}\geqslant 0. \tag{2.5}$$

6. Objective function[1]:

$$\max\{\Phi(y_1,\cdots,y_G)=\Phi(\boldsymbol{y})=\Psi(\varphi_1(\bar{\boldsymbol{y}}_1),\cdots,\varphi_I(\bar{\boldsymbol{y}}_I))\}. \tag{2.6}$$

In the model above k denotes the production process and the corresponding composite product ($k=1,\cdots,K$); g is subscript of product ($g=1,\cdots,G$): i is subscript of group ($i=1,\cdots,I$); G_i is subset of products of group i; j is subscript of a branch ($j=1,\cdots,J$);

1) A detailed discussion of this criterion is given in [21].

K_j denotes the subset of technologies of the branch j; d is subscript of market; D is set of markets with nonconvertible currencies; D' is set of markets with convertible currencies; s is subscript of resource; x_k is the (unknown) intensity of production using technology or the output of the composite product k; y_g is the (unknown) part of final demand for product g; $\bar{\boldsymbol{y}}_i$ is the (unknown) output vector of consumer goods that belong to group i, $\bar{\boldsymbol{y}}_i=\{y_g | g \in G_i\}$, $i=1, \cdots, I$; a_{gk} is the coefficient of input (if $a_{gk}>0$) or output (if $a_{gk}<0$) of product g per unit of composite commodity produced with technology k; f_g is the fixed part of final demand; B_s is the available quantity of primary resources (investments, capacities, natural and labor resources); Δ_d and Δ are the balances of trade in the markets with nonconvertible and convertible currencies respectively; ξ_d is the exchange rate of a convertible currency; c_{gd} and $\hat{c}_{gd}$ are the quantities of exports and imports of commodity g to and from market d respectively; $w_{gd}(c_{gd})$ and $\hat{w}_{gd}(\hat{c}_{gd})$ are the currency earnings and expenditures which are functions of the quantities of exports and imports.

It should be noted that in foreign trade optimization models currency earnings and expenditures are usually assumed to be dependent linearly on quantities of exports and imports [12–20, 24–28]. At the same time the practice of foreign trade indicates that export and import prices do depend on the quantities of exports and imports[2)]; greater quantity is often connected with lower prices. Such a nonlinear relationship is illustrated in fig. 1, where X–axis of the first quadrant shows the quantity of exports and its Y–axis plots export earnings. In the third quadrant quantities of imports and exchange expenditures are plotted respectively.

This nonlinear curve of export earnings and import outlays and quantities of export and import can relate to commodities having high elasticity of exports and imports (with respect to foreign trade prices) and characterised by supply surplus of imported and exported goods over demand. Hence we have a buyer's market: the increase of internal supply of exports or growth of external supply for imports leads to the decrease of export and import prices.

However if a country exports and imports commodities in which demand exceeds supply (seller's market) then a curve assuming increase export earnings and import outlays with growth in the quantities of exports and imports will be more suitable. This curve will show the export and import of raw materials as characterised by a constant increase in demand and a limiting of supply. Such a curve will also denote exports and imports of manufactured goods embodying the most up-to-date technology and therefore having great demand both at home and in external markets. This curve is shown on fig. 2.

The introduction of the nonlinear functions permits us to drop demand and supply constraints on the foreign markets as they are taken into account implicitly in the balances of trade. Indeed these balances incorporate demand and supply of foreign markets by means of the above mentioned interrelation between prices and export returns and import expenditures. Hence, both demand and supply of foreign markets are implicit functions of export and import prices.

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2) The nonlinear relations between production costs of exported and imported goods and the currency earnings (expenditure) in exports (imports) were studied in [29]. Similar curves depict cases when an increase in production costs is accompanied in exports by a decrease in currency earnings or when a contraction of imports increases the production costs of domestic goods.

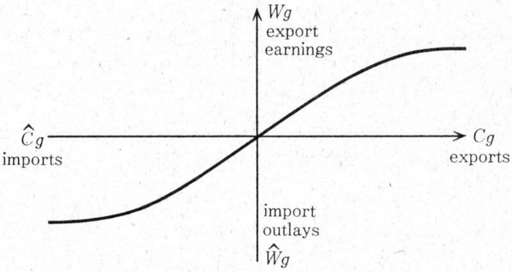

Fig. 1. Relation of export returns w_g and import expenditures $\hat{w}_g$ to quantities of exports c_g and import $\hat{c}_g$ in the case of decreasing export returns and expenditures per unit of exports and imports.

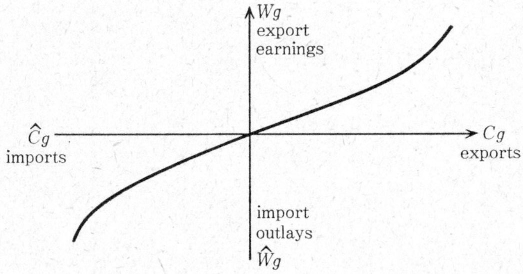

Fig. 2. Relation of the export returns w_g and import expenditures $\hat{w}_g$ to quantities of exports c_g and imports $\hat{c}_g$ in the case of increasing returns and expenditures per unit of exports and imports.

Moreover the information about supply and demand of foreign markets is external for the country optimizing its foreign trade, and hence the accuracy of this data is not adequate. At the same time analysis of the available statistical data and time series of quantities and foreign trade prices permit us to determine the shape and numerical values of the ratios between sales and prices. These ratios, with some adjustments, can be applied to the process of foreign trade optimization for future periods.

Before describing the solution algorithms of the optimization model of production and foreign trade it will be useful to write the model in a compact form. It is achieved with the help of the simplified version of variables:

$$c_{gd}\begin{cases} >0 - \text{exports of commodity } g \text{ to the market } d; \\ <0 - \text{imports of commodity } g \text{ from the market } d; \end{cases}$$

$$w_{gd}(c_{gd})\begin{cases} >0 \text{ with } c_{gd}>0 - \text{volume of exports or currency inflow,} \\ <0 \text{ with } c_{gd}>0 - \text{volume of imports or exchange outflow;} \end{cases}$$

Now the basic model will have the following form:

$$\sum_j \sum_{k \in K_j} a_{gk}x_k + f_g + y_g + \sum_{d \in DUD'} c_{gd} \leqslant 0, \quad g \in G_i, \quad i=1, \cdots, I; \tag{2.7}$$

$$\sum_g w_{gd}(c_{gd}) \geqslant \Delta_d, \quad d \in D; \tag{2.8}$$

$$\sum_{d \in D'} \xi_d \sum_g w_{gd}(c_{gd}) \geqslant \Delta; \tag{2.9}$$

$$\sum_j \sum_{k \in K_j} b_{sk}x_k \leqslant B_s; \tag{2.10}$$

$$x_k \geqslant 0, \quad y_g \geqslant 0, \quad c_{gd} \geqslant 0; \tag{2.11}$$

$$\max \sum_i \Phi_i(\bar{\boldsymbol{y}}_i). \tag{2.12}$$

Let us explain the expressions for balances of trade. Conditions (2. 8) are constraints for net currency expenditures in trade with markets using inconvertible exchanges. That is why the number of such constraints is equal to the number of markets dealing in inconvertible currencies. On the contrary, currency returns in trade with the markets dealing in convertible currencies are described by the constraint (2. 9) as it is possible to sum all convertible currencies with the help of exchange rates.

It is possible here to note one more distinction between our model and other optimization models of production and foreign trade. In the other models coordination of foreign trade with the national economy is carried out by the inclusion of exports and imports in the product balances and in the balances of trade constraints. The dual problem included conditions, where the ratios of foreign trade prices and dual prices of products were taken into account. Our model includes not only currency balances but also net exports (imports) expressed in the dual prices of the product. It will help to make a plan for the national economy more balanced.

3. An Algorithm for the Combined Optimization of the Production and Foreign Trade Problem

The model, described above, can be decomposed in different ways and accordingly one can construct different methods of the iterative coordination of local problems through an aggregated model.

We will show here one such algorithm.

The algorithm includes the following stages.

1. The construction of aggregated indexes and functions

Let p_g, π_d, π and η_s be dual variables corresponding to the constraints (2. 7)–(2. 10). Assume that for iteration σ we receive the next approximations: x_k^σ, y_g^σ, c_{gd}^σ——for solution of the primal problem and p_g^σ, π_d^σ, π^σ, η_s^σ——for solution of the dual problem.

With the help of these approximations we construct the next aggregated indexes and functions.

(a) Aggregated value of inputs-outputs of products of group i for the branch j (aggregation is carried out with the help of shadow prices p_g^σ):

$$a_{ij}^\sigma=\sum_{g\in G_i}\sum_{k\in K_j}p_g^\sigma a_{gk}x_k^\sigma.$$

(b) Aggregated value of the primary resources for output of the branch j:

$$b_j^\sigma=\sum_{k\in K_j}\sum_{s\in S}\eta_s^\sigma b_{sk}x_k^\sigma.$$

(c) Aggregated net exports (imports) value of the products of group i:

$$q_i^\sigma=\sum_{g\in G_i}p_g^\sigma\Big(\sum_{d\in DUD'}c_{gd}^\sigma\Big).$$

(d) Aggregated function of the total net foreign exchange earnings measured in shadow prices π_d^σ, π:

$$w_i^\sigma(u_i)=\sum_{g\in G_i}\sum_{d\in D}\pi_d^\sigma w_{gd}\big(u_i c_{gd}^\sigma\big)+\pi^\sigma\sum_{g\in G_i}\sum_{d\in D'}\xi_d w_{gd}(u_i c_{gd}^\sigma).$$

This function consists of two parts. The first part is the net foreign exchange in trade with the markets of inconvertible currencies. The second part is related to convertible currencies; u_i is the scale of foreign trade transactions compared with the previous iteration.

(e) The total balance of trade for all markets:

$$\delta^\sigma=\sum_{d\in D}\pi_d^\sigma\Delta_d+\pi^\sigma\Delta.$$

(f) Consumption of the products of group i under fixed proportions of products:

$$\varphi_i^\sigma=\Phi_i(v_i\bar{y}_i^\sigma).$$

where v_i is scale of change of consumption vector compared with previous iteration.

(g) Aggregated value of the fixed part of final demand:

$$\tilde{f}_i^{\sigma}=\sum_{g\in G_i} p_g^{\sigma} f_g.$$

(h) Aggregated value of the variable part of final demand:

$$\tilde{y}_i^{\sigma}=\sum_{g\in G_i} p_g^{\sigma} y_g^{\sigma}.$$

(i) Aggregated value of the primary resources input:

$$B^{\sigma}=\sum_{s} \eta_s^{\sigma} B_s.$$

2. The solution of the following aggregated problem, in which we find its minimax point:

$$\min_{\lambda_i,\lambda,\tilde{\lambda}}\max_{z_j,u_j,v_i}\Big\{\sum_i \varphi_i^{\sigma}(v_i)-\sum_i \lambda_i\Big(\sum_j a_{ij}^{\sigma} z_j+\tilde{f}_i^{\sigma}+\tilde{y}_i^{\sigma} v_i+q_i^{\sigma} u_i\Big)-\tilde{\lambda}\Big(\sum_i w_i^{\sigma}(u_i)-\delta^{\sigma}\Big)$$
$$-\tilde{\lambda}\Big(\sum_j b_j^{\sigma} z_j-B^{\sigma}\Big)+M_2\Big[\sum_i(\lambda_i-1)^2+(\tilde{\lambda}-1)^2+(\lambda-1)^2\Big]-M_1\Big(\sum_j(z_j-1)^2$$
$$+\sum_i[(u_i-1)^2+(v_i-1)^2]\Big)\Big\}, \qquad (3.1)$$

subject to

$$\lambda_i>0,\ \lambda>0,\ \tilde{\lambda}>0,\ z_j>0,\ v_i>0,\ u_i>0 \qquad (j=1,\cdots,J;\ i=1,\cdots,I). \qquad (3.2)$$

In this problem v_i, z_j, u_i are scales of changes of the consumption of product i, the output of branch j and the foreign trade activity respectively; λ_i, λ, $\tilde{\lambda}$ are Lagrange multipliers; M_1, M_2 are parameters of the process. Then we can say that in problem (3.1)-(3.2) we find the value of net economic gain. It is calculated as minimum with respect to penalties and maximum with respect to the scales of changes of production, consumption and foreign trade.

Let the solution of this problem be $z_j^{\sigma}, u_i^{\sigma}, v_i^{\sigma}, \lambda_i^{\sigma}, \lambda^{\sigma}, \tilde{\lambda}^{\sigma}$.

3. The calculation of the preliminary approximation for primal and dual variables:

$$\hat{x}_k^{\sigma}=z_j^{\sigma} x_k^{\sigma-1},\ k\in k_j;\ \hat{y}_g^{\sigma}=v_i^{\sigma} y_g^{\sigma-1},\ g\in G_i;\ \hat{c}_{gd}^{\sigma}=u_i c_{gd}^{\sigma-1},\ g\in G_i,\ d\in D\cup D';$$
$$\hat{p}_g^{\sigma}=\lambda_i^{\sigma} p_g^{\sigma-1},\ y\in G_i;\ \hat{\pi}_d^{\sigma}=\tilde{\lambda}^{\sigma}\pi_d^{\sigma-1},\ d\in D;\ \hat{\pi}^{\sigma}=\tilde{\lambda}^{\sigma}\pi^{\sigma-1};\ \hat{\eta}_s^{\sigma}=\lambda\eta_s^{\sigma-1},\ s\in S.$$

The approximations of quantity components (intensities of technologies, volumes of the variable part of the final demand and the net exports (imports)) are calculated using the characteristics of the changes of their scales. Approximations of the shadow prices are calculated by means of the Lagrange multipliers.

4. The formulation of local problems, using the next expressions:

$$\tilde{w}_{gd}^{\sigma}=w_{gd}(\hat{c}_{gd}^{\sigma}),\ d\in D; \qquad (3.3)$$

$$\tilde{w}_g^{\sigma}=\sum_{d\in D'}\xi_d w_{gd}(\hat{c}_{gd}^{\sigma}); \qquad (3.4)$$

$$\tilde{y}_g^{\sigma}=\hat{y}_g^{\sigma}=y_g^{\sigma}+\sum_{d\in D\cup D'}\hat{c}_{gd}^{\sigma}; \qquad (3.5)$$

$$\tilde{a}_{gj}^{\sigma}=\sum_{k\in K_i} a_{gk}\hat{x}_k^{\sigma}; \qquad (3.6)$$

$$\tilde{B}_{sj}^{\sigma}=\sum_{k\in K_j} b_{sk}\hat{x}_k^{\sigma}. \qquad (3.7)$$

4.1. Local problem of the sector of the final consumption and foreign trade for the products of some group i:

$$\max\Big\{\Phi_i(\bar{\boldsymbol{y}}_i)-\sum_{g\in G_i}\Big[\sum_{d\in D}\hat{\pi}_d^{\sigma}\tilde{w}_{gd}+\hat{\pi}^{\sigma}\tilde{w}_g+\hat{p}_g^{\sigma}\tilde{y}_g\Big]$$
$$-\frac{1}{2}Q\sum_{g\in G_i}\Big[\sum_{d\in D}(\tilde{w}_{gd}-\tilde{w}_{gd}^{\sigma})^2+(\tilde{y}_g-\tilde{y}_g^{\sigma})^2+(\tilde{w}_g-\tilde{w}_g^{\sigma})^2\Big]\Big\}, \qquad (3.8)$$

subject to:

$$\tilde{w}_{gd}=w_{gd}(c_{gd}),\quad g\in G_i,\quad d\in D; \tag{3.9}$$

$$\tilde{w}_g=\sum_{d\in D}\xi_d w_{gd}(c_{gd}),\quad g\in G_i; \tag{3.10}$$

$$\tilde{y}_g=y_g+\sum_{d\in D\cup D'}c_{gd},\quad y_g\geqslant 0,\quad g\in G_i, \tag{3.11}$$

where $Q>0$ is a parameter of the algorithm.

Let the solution of this problem be c_{gd}^{σ} and $y_g^{\sigma}(g\in G_i;d\in D\cup D')$.

4. 2. Local problem of the production sector for branch j:

$$\max\left\{\sum_g\hat{p}_g^{\sigma}\tilde{a}_{gi}-\sum_{s\in S}\hat{\eta}_s^{\sigma}\tilde{\beta}_{si}-\frac{1}{2}Q\left[\sum_g(\tilde{a}_{gj}-\tilde{a}_{gj}^{\sigma})^2+\sum_{s\in S'}(\tilde{\beta}_{sj}-\tilde{\beta}_{sj}^{\sigma})^2\right)\right] \tag{3.12}$$

subject to

$$\tilde{a}_{gj}=\sum_{k\in K_j}a_{gk}x_k; \tag{3.13}$$

$$\tilde{\beta}_{sj}=\sum_{k\in K_j}b_{sk}x_k; \tag{3.14}$$

$$x_k\geqslant 0,\quad k\in K_j, \tag{3.15}$$

where

$$\tilde{a}_{gj}^{\sigma}=\sum_{k\in K_j}a_g\hat{x}_{kk}^{\sigma},\quad \tilde{\beta}_{sj}^{\sigma}=\sum_k b_{sk}\hat{x}_k^{\sigma}.$$

Let solution of this problem be x_k^{σ}, $k\in K_j$.

5. Recalculation of the shadow prices:

$$p_g^{\sigma}=\left\{\hat{p}_g^{\sigma}+\alpha\left(\sum_j\sum_{k\in K_j}a_{gk}x_k^{\sigma}+f_g+y_g^{\sigma}+\sum_{d\in D\cup D'}c_{gd}^{\sigma}\right)\right\}_{(+)}, \tag{3.16}$$

$$\pi^{\sigma}=\left\{\hat{\pi}^{\sigma}+\alpha\left(\sum_{d\in D'}\xi_d\sum_g w_{gd}(c_{gd}^{\sigma})-\Delta\right)\right\}_{(+)}; \tag{3.17}$$

$$\pi_d^{\sigma}=\left\{\hat{\pi}_d^{\sigma}+\alpha\left(\sum_g w_{gd}(c_{gd}^{\sigma})-\Delta_d\right)\right\}_{(+)}; \tag{3.18}$$

$$\eta_s^{\sigma}=\left\{\hat{\eta}_s^{\sigma}+\alpha\left(\sum_{k\in K}b_{sk}x_k^{\sigma}-B_s\right)\right\}_{(+)}; \tag{3.19}$$

where α is a parameter of algorithm; subscript $(+)$ denotes: $t_{(+)}=t$, if $t>0$; $t_{(+)}=0$, if $t\leqslant 0$.

The process of the iterative aggregation considered above belongs to the class of the iterative aggregation algorithms using the modified Lagrange function.

For linear problems of such a class the convergence is studied in [21, 22] and for convex problems it is studied in [30, 31]. Though the problem, described here, is a nonconvex one, some theoretical studies of the modified Lagrange function methods concerning nonconvex problems [32, 33] give some ground to suppose that the iterative aggregation methods can be applied to the given nonconvex problem of foreign trade optimization.

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