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A Study on the Effects of Income, Interest Rate and Price Uncertainties upon Optimal Consumption-Saving Decisions*

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1 Introduction

The studies on impacts of changes in income, interest rates and prices upon consumer's optimal decision making have been one of principal issues since the origin of modern economics. They had been long developed through extending the model of consumer's decision making from one commodity model to multicommodity model and/or one period model to multi-period model. They, however, had been regarded as unrealistic because they had neglected uncertainties with respect to income, interest rates and prices in future periods which play important roles in consumer's optimal decision making.

Recently the studies have come to a new stage of development through introducing uncertainties of income, interest rates, and inflation rates into the model of consumer's optimal consumption-saving decisions. The new theory of consumer's choices under uncertainties has modified some of main propositions proved by the traditional theory of consumer's choices and presented many new propositions the traditional theory of consumer's choices could not discuss. One of the most distinguished features of the new theory is that it can analyze effects of changes in uncertainties of income, interest rates and prices on consumer's optimal consumption-saving decisions, which was quitely impossible in the traditional theory.

Most of recent literatures analyzing optimal consumption-saving decisions under uncertainties have mainly concentrated on investigations of the effects of mean preserving changes in uncertainties with respect to income, interest rate and/or price on optimal consumption-saving decisions[1]. They have shown, among other-

things, that, while a mean preserving change in uncertainty of income gives a positive effect on optimal saving of a risk averse consumer, both mean preserving changes in uncertainties of interest rate and price have ambiguous effects on optimal saving of a risk averse consumer. It is necessary to introduce some more strict assumptions in order to determine signs of the effects of interest rate and price uncertainties on optimal consumption-saving decisions[2].

Although the concept of a mean preserving change in uncertainty of a random variable is easily understandable and manageable from the analytical point of view, it has at least a critical defect from the methodological point of view in some economic models where relative economic variables are taken into account. For some relative economic variables can naturally be described in several ways. For example, in the international trade setting with one export good, one import good, and uncertain terms of trade, increases in the uncertainty of trade keeping the expected import price constant (with export price as numeraire) do not keep the expected export price constant (with import price as numeraire). Thus, results of analyses with respect to the effects of a mean preserving change in uncertainty of import (or export)

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1) For the analysis of the effects of mean preserving changes in uncertainty of income and/or interest rate on the consumption-saving decisions, see Anastasopoulos and Kounias [1], Dreze and Modigliani [8], Hahn [10], Hakansson [11], Levhari and Srinivasan [14], Menezes and Auten [16] and Sandmo [20], for example. For the investigation of effect of a mean preserving change in uncertainty of price on saving, see also Anatasopoulos and Kounias [1], and Ishii [13]. And recently Hanson and Menezes [12] and Selden [21] have analyzed the effect of capital risk, using some special properties of preferences.

2) Hahn [10] and others have shown that, if the consumer's relative risk aversion function is less than unity and is nondecreasing, a mean preserving increase in interest rate uncertainty decreases his optimal saving.

price become quite different, depending on which price is chosen as numeraire. The same reasoning will also be applied to the analyses of optimal consumption-saving decisions of a consumer facing uncertainties of prices over multiple periods. Therefore, in this paper we adopt another concept of a change in uncertainty of random variable in order to evade this difficulty.

Diamond and Stiglitz [6] have proposed, as another concept of a change in uncertainty of a random variable which removes the defect mentioned above: the concept of a mean utility preserving change in uncertainty of a random variable. Since this concept of a change in uncertainty of a random variable means a change in uncertainty keeping the mean of utility constant rather than the mean of a random variable itself, it is much harder to understand this conceptually. This concept, however, has an additional advantage from the viewpoint of a practical economic policy. When the government adopts any anti-uncertainty policy which keeps the expected utility of a consumer independent of mean preserving changes in uncertainties of income, interest rate and/or price, then the consumer will face mean utility preserving changes in uncertainties of these random variables[3]. Thus, with increasing chances of the government adopting the anti-uncertainty policies, the mean utility preserving change in uncertainty of a random variable will increase its actual validity.

From the arguments presented above, it seems to us that it would be somewhat meaningful to investigate the effects of mean utility preserving changes in uncertainties of income, interest rate and inflation rate on optimal consumption-saving decisions of a consumer at this stage of development of the new economic theory. The purpose of this paper is to engage in these studies.

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3) For the definition and its implications of a mean utility preserving change in uncertainty of a random variable, see Diamond and Stiglitz [6]. Ishii [13] has explored one of economic policies which keep the consumer's expected utility unchanged from a mean preserving change in uncertainty of the rate of inflation of consumer price. Moreover, the yield-compensated change in uncertainty of security considered by Diamond and Yaari [7] is also an example of a mean utility preserving change in uncertainty.

II　Some Assumptions and Basic Model

Leaving the analysis of interdependence between the length of consumer's planning horizon (or its uncertainty) and his risk aversion functions to other chances, this paper builds a model for investigating the consumer's optimal consumption-saving decisions based on the assumption that the consumer in our model knows his planning periods as n in certainty[4].

Suppose that, at the beginning of t-period within his planning horizon $(t \leq n)$, the consumer faces a problem to devide his initial wealth between consumption and asset holdings. Then his budget constraint in t-period is written as

$$(1) \qquad W_t = P_t c_t + q_t a_t$$

or

$$(1)' \qquad W_t / P_t = c_t + q_t a_t / P_t$$

where W_t stands for the consumer's initial wealth in t-period, c_t consumption in t-period, a_t quantity of asset held in t-period, and P_t and q_t prices of consumer good and asset in t-period, respectively[5].

While the consumer good purchased in t-period is consumed within t-period, the asset is carried over to $t+1$ period and forms the initial wealth W_{t+1} in $t+1$ period, together with noninterest income Y_{t+1} in $t+1$ period;

$$(2) \qquad W_{t+1} = q_{t+1} a_t + Y_{t+1}$$

Taking account of (1) and (2), the real value of initial wealth W_{t+1} in $t+1$ period is given by

$$(3) \quad w_{t+1} = W_{t+1} / P_{t+1}$$
$$= \{ (W_t - P_t c_t) q_{t+1} / q_t + Y_{t+1} \} / P_{t+1}$$

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4) Hakansson [11], Yaari [25] and some others have examined the consumption-saving decisions, taking into consideration the consumer's planning horizon as one of uncertain exogenous variables. It, however, is not always necessary for this paper to regard the consumer's planning horizon as one of uncertain exogenous variables, because the aim of this paper is limited to explore the effects of changes in uncertainty of income, interest rate and rate of inflation on the consumer's optimal decision makings, and the results of our argument do not lose the generality by excluding the uncertainty of planning periods. For the study of the relationship between the consumer's planning horizon and the optimal consumption-saving policy, see Chakravarty [4], Phelps [18] and others.

5) Our model assumes implicitly that the consumer regards a consumer good (or the bundle of their goods) and an asset (or the bundle of assets) as quite different ones.

Here, normalizing all nominal variables in terms of P_t and expressing rates of inflation of P_t and q_t in t period as π_t and r_t and growth rate of Y_t in t_i' period as g_t respectively, (3) is expressed as

(3)$'$ $w_{t+1} = \{(w_t - c_t)(1 + r_t) + Y_t(1 + g_t)\}/1 + \pi_t$

In what follows, the rate of inflation of asset price is called as the rate of interest or interest rate[6].

It is assumed that, though π_t, r_t and g_t are all unknown to the consumer at the beginning of t period, the conditional density function of these random variables given those values before t period, Π_{t-1}, R_{t-1} and G_{t-1}, i. e.

(4) $\phi_t(g_t, r_t, \pi_t | G_{t-1}, R_{t-1}, \Pi_{t-1})$

is known to him certainly.

The consumer's preference function of n-period consumptions is given by

(5) $$V = \sum_{i=1}^{n} \beta^{i-1} u(c_i)$$

where β is the discount factor with $0 < \beta < 1$. The instantaneous utility function $u(c_i)$ in i period is assumed to satisfy the theorem of expected utility function proposed by von Neuman and Morgenstern [23] and to be a thrice differentiable one with usual properties of

(6) $u'(c_i) > 0, \quad u''(c_i) < 0$

In arguments under uncertainty, $u''(c_i) < 0$ implies that the consumer is risk averse.

Since rates of interest and inflation and growth rate of income in future periods are all random variables, all consumptions in future periods are also random variables. Therefore, the consumer in our model is supposed to make optimal consumption-saving decisions so as to maximize his expected utility function, i. e.

(7) $$EV = \sum_{i=1}^{n} \beta^{i-1} \underset{i}{E}[u(c_i)]$$

subject to the stochastic budget constraints(3)$'$, where $\underset{i}{E}[\ ]$ is an expectation operator of conditional random variables π_i, r_i and q_i given their values until $i-1$ periods.

Although there are several ways to solve the optimization problem of the consumer in our model, we use a procedure of dynamic programming in this paper. So, define $f_k(w)$ as the present value of expected utilities which is obtained by starting the present period with

initial wealth w and adopting optimal consumption-saving policy for k periods. Then, the problem in this section is formulated as

(8) $f_n(w_t) = \underset{0 \le c_t \le w_t}{\text{Max.}} [u(c_t) + \beta E[f_{n-1}(w_{t+1})]]$

with

$$f_1(w_t) = \underset{0 \le c_t \le w_t}{\text{Max.}} u(c_t)^{[7]}.$$

If $n = 2$, the problem becomes an usual two-period optimization problem which determines c_t so as to maximize

$u(c_t) + E[u(\{(w_t - c_t)(1 + r_t) + Y_t(1 + g_t)\}/1 + \pi_t)]$

(8) is essentially similar to the formulation in Phelps [18] except that(8) additionally includes the rate of inflation of consumer price, and it is also similar to one in Anastasopoulos and Kauniaus[1] except that(8) does not take account of any loans.

In following sections we drop the subscript t which expresses the period of variables treated in nonconfusing cases, because t period is regarded as the present period in this paper. Thus it should be noted that the optimal value of consumption which is obtained by solving (8) is the optimal consumption in the present period that is chosen by the consumer having initial wealth w_t and planning periods $n^{[8]}$.

III Intertemporal Risk Aversion Functions

Arrow [2] and Pratt [19] have defined, as measures of risk aversion of an economic unit whose utility function is expressed as $u(Z)$, the absolute and relative risk aversion functions as

$R_A = -u''(Z)/u'(Z)$ and $R_R = -Zu''(Z)/u'(Z)$,

respectively. In this paper these risk aversion functions are called the stationary risk aversion functions because of the stationary features of the utility function and its variable. Additionally Sandmo [20], assuming the expected

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6)   If there exists any dividend on asset holding, the rate of interest would be defined so as to include it.

7)   The theory of dynamic programming teaches that, if the instantaneous utility function $u(c_i)$ has properties shown by(6) and is thrice differentiable, the indirect expected utility function $f_n(w)$ is also thrice differentiable and has features of $f_n'(w) > 0$ and $f_n''(w) < 0$, which also hold in the case of corner solutions. For the proof of this kind of theorem, see the chapter V of Bellman [3] and the chapter 5 of White [24], for example.

8)   It is shown that the solution of (8) exists uniquely. The proof, however, is omitted because it is essentially similar to those in the existing literatures.

utility function which is maximized by a consumer in a two-period model under uncertainty as $E[u(c_1, c_2)]$, has proposed the intertemporal absolute and relative risk aversion functions as

$$R_A^2 = -u''(c_1, c_2)/u'(c_1, c_2) \text{ and}$$
$$R_R^2 = -c_2 u''(c_1, c_2)/u'(c_1, c_2)$$

respectively, where $u'(c_1, c_2) = \partial u(c_1, c_2)/\partial c_2$ and $u''(c_1, c_2) = \partial^2 u(c_1, c_2)/\partial c_2^2$. We also define the risk aversion functions of the consumer with n planning periods, following the definitions of those by Arrow[2], Pratt[19] and Sandmo[20][9].

In our model the objective function to be maximized by the consumer is given by

$$u(c_t) + \beta E[f_{n-1}(w_{t+1}(c_t))]$$

Therefore it is quite clear from this formulation that a random variable is w_{t+1} and that the function including the random variable is f_{n-1} ( ). Thus, in much the same reasonings as those by Arrow, Pratt and Sandom have been defined, the intertemporal absolute and relative risk aversion functions of the consumer with n planning periods are defined as

$$(9) \quad R_A^n = -f_{n-1}''(w_{t+1})/f_{n-1}'(w_{t+1}),$$
$$R_R^n = -w_{t+1}f_{n-1}''(w_{t+1})/f_{n-1}'(w_{t+1})$$

respectively. (9) shows that the n period intertemporal risk aversion functions depend not only on the consumer's initial wealth in future period but also on his planning horizon[10].

Generally there are some relations between the intertemporal risk aversion functions defined by (9) and the stationary ones defined by Arrow and Pratt. They are shown by the next proposition:

Proposition I Suppose that the consumer is a representative one in the sense that he always chooses interior optimal consumptions. Then

$$-\frac{f_k''(w)}{f_k'(w)} = -\frac{u''(c_k)}{u'(c_k)} \cdot M_c^k \text{ and}$$

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9) Sandmo [20] has not used superscripts 2 in defining his risk aversion functions. In this paper it is introduced by the author for expressing the number of planning periods. Therefore, the risk aversion functions of consumer with n planning periods are generally expressed as R_A^n and R_R^n, respectively. For other types of risk aversion functions, see Menezes and Hanson [12] and Zeckhausen and Keeler [26], for example.

10) The investigation whether the intertemporal risk aversion functions are increasing or decreasing with respect to the consumer's planning periods is also one of interesting and important economic problems.

$$-w \frac{f_k''(w)}{f_k'(w)} = -c_k \frac{u''(c_k)}{u'(c_k)} \cdot \eta_c^k$$

hold for any $k \geq 2$, where M_c^k and η_c^k are respectively marginal propensity to consume and elasticity of consumption in the present period with respect to initial wealth of the consumer with k planning periods.

(proof) In the case $n = k$, the interior optimal consumption c^k is obtained by solving

$$(10) \quad u'(c^k) - \beta E[f_{k-1}'(w_{k+1})$$
$$(1+r_k)/(1+\pi_k)] = 0$$

where $w_{k+1} = \{(w_k - c_k)(1+r_k) + Y_k(1+g_k)\}/(1+\pi_k)$. On the other hand,

$$(11) \quad f_k(w) = u(c_k) + \beta E[f_{k-1}(w_{k+1})]$$

holds from the definition of $f_k(w)$. Therefore, differentiating the both sides of (11) with respect to w, we get

$$(12) \quad f_k'(w) = \beta E[f_{k-1}'(w_{k+1})(1+r_k)/(1+\pi_k)]$$

It follows that (10) and (12) yield

$$(13) \quad f_k'(w) = u'(c_k)$$

And, once again, differentiating the both sides of (13) with respect to w, we obtain

$$(14) \quad f_k''(w) = u'(c_k) \cdot \partial c_k/\partial w$$

Thus, defining the intertemporal absolute and relative risk aversion functions from (13) and (14) respectively, the proposition is immediately derived. QED

Since it is proved from (10) that $\partial c^k/\partial w$ is positive and less than unity, the next corollary is easily derived from the proposition I:

Corollary I For any $k \geq 2$,

$$R_A^k < R_A$$

where R_A is the stationary absolute risk aversion function defined by using the instantaneous utility function $u(c_k)$.

Though the analysis in the next section needs to make clear whether the intertemporal absolute risk aversion function is increasing, constant or decreasing, it is impossible to show definitely which is more plausible without introducing additional assumptions. We, however, do not introduce any additional assumptions until the final section. Therefore, we will end this section with proposing the next lemma which is used in proving our main propositions in the next section.

Lemma I If $R_A^n(w)$ is decreasing (constant or increasing) for all n, then

$$f_k' f_k''' - (f_k'')^2 > (= \text{ or } <) 0$$
$$\text{for any } k \geq 1.$$

The proof of this lemma is omitted because it is immediately obtained from the definition of $R_A^k(w)$.

IV The Effects of Changes in Uncertainties of Income, Interest Rate and Rate of Inflation on Optimal Consumption-Saving Decisions

It is also one of the important purposes of economics to canvass the effects of changes in uncertainties of g_t, r_t and π_t on the optimal consumption c_t^*. In this paper a change in uncertainty of a random variable is measured by the mean utility preserving change in its uncertainty.

Let us define newly the expected utility function as

$$(15) \qquad \int U(\theta, \alpha) \, \mathrm{dF}\,(\theta, \gamma)$$

where $U(\ )$ is a thrice differentiable utility function which includes a random variable θ and a control one α, and $F(\theta, \gamma)$ is a distribution function of θ where γ stands for an index of mean utility preserving change in uncertainty of θ. Then, from the Theorem 2 in Diamond and Stiglitz[6], the next lemma is easily derived:

Lemma II The control variable $\alpha * (\gamma)$ which maximizes the expected utility function $\int U(\theta, \alpha) \, \mathrm{dF}\,(\theta, \gamma)$ is an increasing (a decreasing) function of γ in the case $U_\theta U_{\alpha\theta\theta} - U_{\alpha\theta} U_{\theta\theta} > (<) 0$[11].

The proof of this lemma is omitted because it is similar to that of the Theorem 2 in Diamond and Stiglitz except that they have used $(U_\theta U_{\alpha\theta\theta} - U_{\alpha\theta} U_{\theta\theta}) / U_\theta$ instead of $U_\theta U_{\alpha\theta\theta} - U_{\alpha\theta} U_{\theta\theta}$ because they have assumed $U_\theta > 0$. It, however, is clear that $U_\theta > 0$ does not always hold in general. Therefore, we have rewritten their theorem as lemma II for more generalized analyses.

Now we turn our attention to our main purpose of analyzing the effects of changes in uncertainties of random variables on optimal consumption decisions. The followings are the comparative statistics which examine the effect of change in uncertainty of a random variable on optimal consumption in the case when other random variables are fixed at their expected values respectively.

As regards the effect of a change in uncertainty of the growth rate of income g_t on optimal consumption in t period c_t^*, the next

proposition hold:

Proposition II If the intertemporal absolute risk aversion function is decreasing (increasing), the optimal consumption in the present period decreases (increases) with a mean utility preserving increase in uncertainty of the growth rate of income g, and *vice versa*.

(proof) Let us denote c_t which maximizes the right hand side of (8) by c_t^*. Then, from its definition, $f_n(w_t)$ is given by

$$(16) \quad f_n(w_t) = u(c_t^*) + \beta E[f_{n-1} \\ (\{(w_t - c_t^*)(1 + r_t) + Y_t(1 + g_t)\} / 1 + \pi_t)]$$

Since, in this case, that g_t and c_t^* are only a random variable and a control variable respectively, g_t and c_t^* in (16) correspond to θ and α^* in (15), respectively. Therefore, regarding the $f_{n-1}(\ )$ function in (16) as the $U(\ )$ function in (15), we can obtain $U_\theta U_{\alpha\theta\theta} - U_{\alpha\theta} U_{\theta\theta}$ in this case as

$$(17) \quad U_\alpha U_{\alpha\theta\theta} - U_{\alpha\theta} U_{\theta\theta} \\ = -(1 + r_t) Y_t^3 \{f_{n-1}' f_{n-1}''' - (f_{n-1}'')^2\}$$

On the other hand, the lemma I has shown that a decreasing (increasing) intertemporal absolute risk aversion function is sufficient for $f_{n-1}' f_{n-1}''' - (f_{n-1}'')^2 > (<) 0$. Thus, lemma I-II and (17) yield the proposition II.

QED

It is also easily shown from (16) that, if the intertemporal absolute risk aversion function is decreasing (increasing), the optimal consumption in the present period decreases (increases) with a mean preserving increase in uncertainty of growth rate of income[12]. Consequently, this and our proposition II imply that a change in uncertainty of growth rate of income gives a negative effect on the optimal consumption regardless of the difference of measures with respect to its uncertainty. Moreover these propositions propose one of theoretical foundations of the Friedman's permanent income hypothesis in consumption decisions.

Next, we consider the effect of a mean utility preserving change in uncertainty of interest rate on the optimal consumption in the present period. In this case, since only the interest rate is a random variable, c_t^* and r_t in

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11) Here $U_\theta = \partial U(\theta, \alpha) / \partial \theta$, $U_{\alpha\theta} = \partial^2 U(\theta, \alpha) / \partial \theta \partial \alpha$, $U_{\theta\theta} = \partial^2 U(\theta, \alpha) / \partial \theta^2$ and so on.

12) Sandmo [20] and others have proved that the optimal consumption decreases with a mean preserving increase in uncertainty of income (or its growth rate) under the assumption of decreasing intertemporal absolute risk aversion function.

(16) correspond to α^* and θ in (15), respectively. Accordingly, in the same way as (17) has derived, we get $U_\theta U_{\alpha\theta\theta} - U_{\alpha\theta} U_{\theta\theta}$ in this case as

(18)　$U_\theta U_{\alpha\theta\theta} - U_{\alpha\theta} U_{\theta\theta} = -(w_t - c_t^*)^3 (1+r_t)$
　　　　$\{f_{n-1}' f_{n-1}''' - (f_{n-1}'')^2\}$

Since $-(w_t - c_t^*)^3 (1+r_t)$ is negative in general, the next proposition is proved in much the same reasoning as the proposition II has been established.

Proposition III　A mean utility preserving increase in uncertainty with respect to the rate of interest decreases (increases) the optimal consumption in the present period if the consumer's intertemporal absolute risk aversion function is decreasing (increasing), and *vice versa*.

As is well known the sign of the effect of mean preserving change in uncertainty with respect to interest rate is not definitely judged in general without setting somewhat more strict assumptions than existing ones on the risk aversion functions[13]. On the other hand, the proposition III tells us that the sign of the effect of a mean utility preserving change in uncertainty of interest rate on optimal consumption is judged definitely on only the feature that if the consumer's intertemporal absolute risk aversion function is decreasing (constant or increasing). The arguments that the intertemporal absolute risk aversion function is decreasing under some plausible assumptions are presented in the next section.

Finally, for the effect of a mean utility preserving change in uncertainty of the rate of inflation of consumer price on the optimal consumption in the present period, the next proposition holds:

Proposition IV　A decreasing (increasing) intertemporal absolute risk aversion function is sufficient for the optimal consumption in the present period to increase (decrease) with a mean utility preserving increase in uncertainty of the rate of inflation of consumer price, and *vice versa*.

(proof) Note that c_t^* and π_t in (16) respectively correspond to α^* and θ in (15). Then, through a direct calculation we obtain

(19)　$U_\theta U_{\alpha\theta\theta} - U_{\alpha\theta} U_{\theta\theta}$
　　　　$= \dfrac{(1+r_t) Z^3}{(1+\pi_t)^6} \{f_{n-1}' f_{n-1}''' - (f_{n-1}'')^2\}$

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13)　For example, see Hahn[10].

where $Z = \{(w_t - c_t^*)(1+r_t) + Y_t(1+g_t)\}/1+\pi_t > 0$.

Lemma I and (19) yield

(20)　　　$U_\theta U_{\alpha\theta\theta} - U_{\alpha\theta} U_{\theta\theta} > (<) 0$

if the consumer's intertemporal absolute risk aversion function is increasing (decreasing). Thus, from lemma II and (20), the assertion follows.　　　　　　　　　　　　　　QED

As is clear from the definition, a change in uncertainty of the rate of inflation of consumer price also implies a change in uncertainty of the real value of initial wealth in future periods. Therefore, it is expected intuitively that any change in uncertainty with respect to the rate of inflation of consumer price has a significant effect on the optimal consumption decisions. The effect, however, had not been investigated appropriately until the proposition IV has been presented.

The proposition IV has two economic implications. One is, in its literal sense, that the optimal consumption is a decreasing (an increasing) function of uncertainty with respect to the rate of inflation of consumer price, if the consumer has an increasing (a decreasing) intertemporal absolute risk aversion function. And the other is shown as followings in the relation with the proposition III. That is, it is clear from (18) and (19) that, while the definition of real interest rate as $(r_t - \pi_t)$ is not appropriate in analyzing the absolute value of effect of a mean utility preserving change in uncertainty of the real rate of interest on optimal consumption, the definition of real rate of interest isn't inappropriate in investigating the sign of the effect. As a result of this argument, we obtain the next proposition:

Proposition V　If the consumer's intertemporal absolute risk aversion function is decreasing (increasing), a mean utility preserving increase in uncertainty of real rate of interest decreases (increases) the optimal consumption in the present period, and *vice versa*.

V　Concluding Remarks

In this paper we have analyzed the effects of mean utility preserving changes in uncertainties of growth rate of income, interest rate and rate of inflation on the optimal consumption decisions of the risk averse consumer without setting any additional assumptions with respect to his intertemporal absolute risk aversion function. However, it is shown that

the consumer's intertemporal absolute risk aversion function is decreasing under the two plausible assumptions of decreasing stationary absolute risk aversion function and of decreasing marginal propensity to consume.

From the proposition I,

(21) $R_A{}^{n+1}(w) = R_A(c_n(w)) \cdot \partial c_n / \partial w$

holds between the intertemporal and stationary absolute risk aversion functions, $R_A{}^{n+1}(w)$ and $R_A(c_n)$. Thus, differentiating the both sides of (21), we have

$$(22) \quad \frac{\partial R_A{}^{n+1}(w)}{\partial w} = \frac{\partial R_A(c_n)}{\partial c_n} \cdot \left(\frac{\partial c_n}{\partial w}\right)^2 + R_A \cdot \left(\frac{\partial^2 c_n}{\partial w^2}\right)$$

Since the assumptions of decreasing stationary absolute risk aversion function and decreasing marginal propensity to consume mean $\partial R_A / \partial c_n < 0$ and $\partial^2 c_n / \partial w^2 < 0$ respectively, it is shown easily, substituting these results into the right hand side of (22), that the intertemporal absolute risk aversion function is decreasing, i. e. $\partial R_A{}^{n+1} / \partial w < 0$. Consequently, if the assumptions of decreasing stationary absolute risk aversion function and decreasing marginal propensity to consume are plausible ones as some economists have argued, the propositions II-IV are rewritten more definitely as

Proposition VI   The optimal consumption in the present period decreases with mean utility preserving increases in uncertainties with respect to the growth rate of income and the (real) rate of interest, and it increases with a mean utility preserving increase in uncertainty of the rate of inflation of consumer price, and *vice versa*.

In the section I, we have already refered to one of economic significant implications of analyzing the effects of mean utility preserving changes in uncertainties of random variables on the optimal consumption decisions. The economic implications of mean utility preserving changes in these uncertainties, however, have not been made clear as much as those of mean preserving changes in the uncertainties have been, which is one of the reasons why, in analyzing the effects of uncertainties on the optimal consumption decisions, the analytical attention has been concentrated much more on mean preserving changes in uncertainties than on mean utility preserving changes in uncertainties. But the significance of examining the effects of mean

utility preserving changes in uncertainties on the optimal consumption decisions will increase with the development of studies on them.

There are many ways by which the model in this paper can be extended and generalized furthermore. One of them which seems most important to us is to treat the planning horizon of consumer as one of endogeneous variables. It is generally said that the planning horizon of an unmarrid young man is much shorter than that of a marrid middle-aged man with some childeren. If this statement is true, the usual assumption that the consumer's planning horizon are equal to his life-time can not insist on its plausibility any longer. Then, the consumer's planning horizon become quite different ones from the horizon of his life-time and are regarded as one of endogeneous variables which depend on his life-time remained, the initial wealth, the degree of risk aversion and so on.

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References

[1] Anastasopoulos, A and S. Kounias, "Optimal Consumption Over Time When Prices and Interest Rates Follow a Markovian Process," *Econometrica*, 45 (March 1975), 261–281.

[2] Arrow, K. J., "Exposition of The Theory of Choice under Uncertainty," in *Essays in the Theory of Risk-Bearing*, North-Holland, 1970, 44–89.

[3] Bellman, R. E., *Dynamic Programming*, sixth printing, Princeton University Press, 1972.

[4] Chakravarty, S, "Optimal Saving with Finite Planning Horizon," *International Economic Review*, 3 (Sep. 1962), 338–355.

[5] Deaton, A. S., "Wealth Effects on Consumption in a Modified Life Cycle Model," *Review of Economic Studies*, 39 (Oct. 1972), 443–453.

[6] Diamond, R. A. and J. E. Stiglitz, "Increases in Risk and in Risk Aversion," *Journal of Economic Theory*, 8 (July 1974), 337–360.

[7] Diamond, P. A. and M. E. Yaari, "Implications of a Theory of Rationing for Consumer Choice under Uncertainty," *American Economic Review*, 62 (June 1972), 333–343.

[8] Dreze, J. H. and F. Modigliani, "Consumption Decisions under Uncertainty," *Journal of Economic Theory*, 5 (Dec. 1972), 308–335.

[9] Fisher, S., "A Life Cycle Model of Life Insurance Perchases," *International Economic Review*, 14 (Feb. 1973) 132–153.

[10] Hahn, F. H., "Savings and Uncertainty,"

Review of Economic Studies, 37 (Jan. 1970), 21–24.

[11] Hakansson, N. H., "Optimal Investment and Consumption Strategies Under Risk, An Uncertain Lifetime, and Insurance," *International Economic Review*, 10 (Oct. 1969), 443–466.

[12] Hanson, D. L. and C. F. Menezes, "The Effect of Capital Risk on Optimal Saving Decisions," *Quarterly Journal of Economics*, 92 (Nov. 1978), 653–670.

[13] Ishii, Y., "On The Theory of Consumption-Saving Policy under Interest Rate, Income and Price Uncertainties," in *The Fundamental Theories of Saving and Money*, ed. by the Saving Promoting Commission, Tokyo, 1976, 51–104.

[14] Levhari, D. and J. Srinivasan, "Optimal Savings under Uncertainty," *Review of Economic Studies*, 36 (April 1969),153–163.

[15] Menezes, C. F. and D. L. Hanson, "On the Theory of Risk Aversion," *International Economic Review*, 11 (Oct. 1970), 481–487.

[16] Menezes, C. F. and G. E. Auten, "The Theory of Optimal Saving Decisions under Income Risk," *International Economic Review*, 19 (Feb. 1978), 253–258.

[17] Mirman, L. J., "Uncertainty and Optimal Consumption Decisions," *Econometrica*, 39 (Jan. 1971), 179–185.

[18] Phelps, E. S. "The Accumulation of Risky Capital: A Sequential Utility Analysis," *Econometrica*, 30 (Oct. 1962), 729–743.

[19] Pratt, J. W., "Risk Aversion in the Small and in the Large," *Econometrica*, 32 (Jan.-April 1964), 122–136.

[20] Sandmo, A. "Capital Risk, Consumption, and Portfolio Choice," *Econometrica*, 37 (Oct. 1969), 586–599.

[21] Selden, L., "An OCE Analysis of The Effect of Uncertainty on Saving under Risk Preference Independence," *Review of Economic Studies*, 46 (Jan. 1979), 73–82.

[22] Stiglitz, J. E. and M. Rothschild, "Increasing Risk: A Definition," *Journal of Economic Theory*, 2 (Sep. 1970) 225–243.

[23] von Neuman, J. and O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton University Press, 1953.

[24] White, D. J., *Dynamic Programming*, Oliver & Boys, 1969.

[25] Yaari, M. E., "Uncertain Lifetime, Life Insurance and the Theory of the Consumer," *Review of Economic Studies*, 32 (April 1965), 137–150.

[26] Zeckhausen, R. and E. Keeler, "Another Type of Risk Aversion," *Econometrica*, 38 (Sep. 1970), 661–665.