Note on Technical Changes and the Wage-Profit Curve*

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1. This note is to show some of the results by Roemer [5] in a simple and unified manner and also to give new results concerning technical changes in terms of the wage-profit curve. That is, we first show that if the profit rate is fixed, the real wage rate goes up after the adoption of cost-reducing technical changes. Next we prove that if technical changes are cost-reducing at some profit rate and of the capital-using-labour-saving type, then they also reduce the labour value of each commodity. Lastly, these propositions are summarized using the wage-profit curve.

Let us explain our model and symbols. For the sake of simplicity, use is made of a Leontief model of circulating capital, in which there are n commodities and n industries. Symbols are as follows:

$\boldsymbol{A}$: input coefficient matrix (n by n),
$\boldsymbol{I}$: identity matrix (n by n),
$\boldsymbol{L}$: labour input row n-vector,
$\boldsymbol{C}$: column n-vector of standard consumption basket for workers,
$\boldsymbol{p}$: price row n-vector (labour as numeraire),
$\boldsymbol{\Lambda}$: value row n-vector,
r: rate of profit,
w: real wage rate.

In this note, a prime attached to a variable indicates time-derivative d/dt. But this need not be regarded as requiring literally the differentiability of variables: rather it should be understood as the ratio of finite differences $\Delta/\Delta t$, neglecting small numbers of the second order. In fact our arguments below could be carried out using finite differences but differentials have been adopted on account of simplicity in presentation. Inequality signs for vector comparison are used in a usual sense (See Nikaido [3, p. xii]).

We make the following assumptions.

A1. The matrix $\boldsymbol{A}$ is indecomposable, and $\boldsymbol{L}$ and $\boldsymbol{C}$ are semi-positive.

A2. The matrix $\boldsymbol{A}$ is productive so that the maximum rate of profit is positive.

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In this note we reserve the phrase 'technological' changes, regarding these as more significant ones, e. g., the former includes new commodities than technical and hence completely new processes.

2. In our model the price equation is:

$$p = (1+r)(\boldsymbol{pA}+\boldsymbol{L}), \text{ or}$$
$$p = (1+r)\boldsymbol{L}(\boldsymbol{I}-(1+r)\boldsymbol{A})^{-1}, \tag{1}$$

while the value equation is:

$$\boldsymbol{\Lambda} = \boldsymbol{\Lambda}A+\boldsymbol{L}, \text{ or}$$
$$\boldsymbol{\Lambda} = \boldsymbol{L}(\boldsymbol{I}-\boldsymbol{A})^{-1}.$$

The real wage rate is defined as:

$$w \equiv 1/\boldsymbol{pC}.$$

Units of commodities are chosen so that $\boldsymbol{pC}=1$ before technical changes. Now suppose that technical changes have taken place and are adopted because they are cost-reducing. In our model technical changes are expressed as changes along with time in the coefficients of $\boldsymbol{A}$ and $\boldsymbol{L}$. We fix the profit rate at r° less than the maximum rate of profit ($r^\circ>0$). Let

$$\boldsymbol{D} \equiv (\boldsymbol{I}-(1+r^\circ)\boldsymbol{A})^{-1}.$$

Then, from equation (1) we have

$$\begin{aligned}\boldsymbol{p}' &= (dp_1/dt, \cdots, dp_n/dt)\\ &= (1+r^\circ)\boldsymbol{L}'\boldsymbol{D}-(1+r^\circ)\boldsymbol{LD}(-(1+r^\circ)\boldsymbol{A}')\boldsymbol{D}\\ &= (1+r^\circ)(\boldsymbol{pA}'+\boldsymbol{L}')\boldsymbol{D},\end{aligned}$$

using the relation $(\boldsymbol{M}^{-1})'=-\boldsymbol{M}^{-1}\cdot\boldsymbol{M}'\cdot\boldsymbol{M}^{-1}$, where $\boldsymbol{M}$ is an arbitrary n by n regular matrix (See Appendix 1). Therefore, it follows that

$$\begin{aligned}w' &= -\boldsymbol{p}'\boldsymbol{C} \text{ (because } \boldsymbol{pC}=1)\\ &= -(\boldsymbol{pA}'+\boldsymbol{L}')\boldsymbol{x}^*,\end{aligned} \tag{2}$$

where $\boldsymbol{x}^*$ is the Golden Age activity level vector defined by solving the equation

$$\boldsymbol{x}^* = (1+r^\circ)(\boldsymbol{Ax}^*+\boldsymbol{C}).$$

Note that $\boldsymbol{x}^*$ is not an actual activity level vector but merely an operational one. Cost-reducing technical changes in all branches here mean

$$\boldsymbol{pA}'+\boldsymbol{L}' \leq 0.$$

Thus we have

$$\boldsymbol{p}' < 0,$$

since $\boldsymbol{D}>0$ because of the indecomposability of $\boldsymbol{A}$. Next we obtain

$$w' > 0.$$

The size of the increment in the real wage rate is the weighted sum of reduced costs of each industry, the weights being the Golden Age activity levels. The above result is formally stated as Proposition 1.

Proposition 1. If the profit rate is fixed, the real wage rate goes up after the adoption of cost-reducing technical changes.

This proposition is dual to one of the Okishio's results in [4, p. 113]: if the real wage rate is fixed, the profit rate goes up after the adoption of cost-reducing technical changes (See also Roemer [5, p. 409], Nakatani [2, p. 73]).

Fig. 1

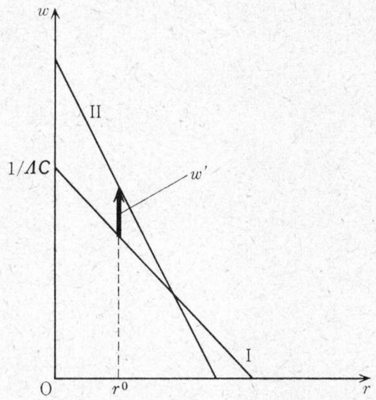
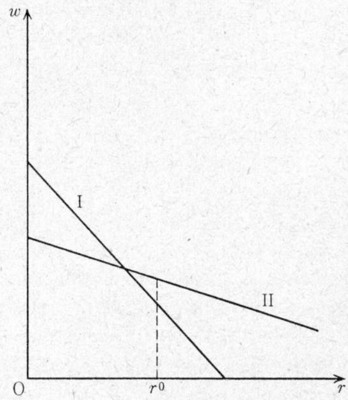


Fig. 2



3. What will happen to labour values?

$$\Lambda' = L'(I-A)^{-1} - L(I-A)^{-1} \cdot (-A') \cdot (I-A)^{-1} = (\Lambda A' + L')(I-A)^{-1}. \quad (3)$$

Suppose technical changes are of the capital-using-labour-saving type (CU-LS type), i.e., $A' \geq 0$ and $L' \leq 0$. Then $pA' + L' \leq 0$ implies $\Lambda A' + L' \leq 0$ because $0 < \Lambda < p$. So we have from equation (3)

$$\Lambda' < 0$$

since $(I-A)^{-1} > 0$. This result is summarized as Proposition 2.

Proposition 2. If technical changes are cost-reducing at $r°$ and of the CU-LS type, then they also reduce the labour value of each commodity. This proposition is due to Roemer [5, p. 411].

4. We investigate the above phenomena in terms of the wage-profit curve. Now suppose there are two sets of production processes, I and II, each of which defines the wage-profit curve as is depicted in Fig. 1. In such a case, that is, the curve II is always above the curve I when the profit rate is less than $r°$, we say the set II is 'more gainful' than the set I for the range of profit rate $[0, r°]$. We show

Proposition 3. If technical changes are cost-reducing at $r°$ and of the CU-LS type, then the production processes become more gainful than the original processes for the range of profit rate $[0, r°]$.

This proposition is natural and comprehensible. Our proof goes like this. Take a profit rate r^* less than the original rate $r°$, and denote by p^* the equilibrium price vector corresponding to r^*. At this r^*, technical changes are still cost-reducing, i.e., $p^*A' + L' \leq 0$, since $0 < p^* < p$. And so, the real wage can increase at r^* by Proposition 1.

Notice that in the case of Proposition 3, the maximum rate of profit becomes smaller because the Frobenius root of A becomes greater. On the other hand, there exists the possibility of multiple switching between the two curves, time 0 and Δt, to the right of profit rate $r°$. As is easily proven likewise, however, if technical changes occur in only one industry, then there can be no double switching (See Appendix 2).

5. Proposition 3 implies that if technical changes are cost-reducing but not of the CU-LS type, then there may be the case as is depicted in Fig. 2. Thus, a comment is in order concerning the Morishima-Catephores' result in [1, p. 111], which asserts that under reasonable assumptions the tail of the new frontier lies above the corresponding part of the old one. At first sight, the case of Fig. 2 seems to damage the Morishima-Catephores' result. But this is not so. For, their assumption made in the middle of the footnote of [1, p. 111] is required for any arbitrarily fixed value of the real wage rate and thus it is rather restrictive. On the other hand, we require the information about cost-reduction only at one particular rate of profit.

6. We can prove in a similar way the result by Roemer [5, p. 411] for the case of capitalsaving-labour-using technical changes. The results in this note can be generalized into a Leontief model with durable capital goods which exhibit quantitative depreciation.

Since we have made clear the size of increment in the real wage rate after technical changes when the profit rate is fixed, we know from this that the rate of profit must fall if the real wage rate increases beyond that size.

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Appendix

1. $\boldsymbol{M}\cdot\boldsymbol{M}^{-1}=\boldsymbol{I}$. Differentiate both sides and we have

$$\boldsymbol{M}'\cdot\boldsymbol{M}^{-1}+\boldsymbol{M}(\boldsymbol{M}^{-1})'=0.$$

From this it follows that

$$(\boldsymbol{M}^{-1})'=-\boldsymbol{M}^{-1}\cdot\boldsymbol{M}'\cdot\boldsymbol{M}^{-1}.$$

2. Let us write down again equation (2) in the text.

$$w'=-(\boldsymbol{p}\boldsymbol{A}'+\boldsymbol{L}')\boldsymbol{x}^*.$$

When $r°$ increases, the elements of $\boldsymbol{p}$ and $\boldsymbol{x}^*$ increase disproportionately. So the sign of w' may change more than once. Here, suppose technical changes occur only in the i-th industry. Then, once $\{\boldsymbol{p}(\boldsymbol{A}')_i+\boldsymbol{L}_i'\}\boldsymbol{x}_i^*$ becomes positive at a certain r, it is positive when the profit rate is greater than the r.

References

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