Demand Uncertainty and the Theory of Investment*

——A Reinterpretation of Theories of Investment——

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ABSTRACT

In this paper we examine at first fixed investment behavior under uncertain demand, showing that the firm will determine its investment at such a level that the expected rate of profits becomes equal to the rate of discount of risk, which depends on its attitude towards risk and a random demand exogenously given. Then, basing on this analysis, we reinterpretate choice-theoretically theories of investment, especially Acceleration Principle of Harrod type, Profit Principle of Kalecki–Kaldor type and Keynes' theory of investment.

1 Introduction

Examinations of investment behavior of the firm under uncertainty have been so far concentrated on what effects the existence of and/or the increase in uncertainty about output price, prices of inputs, demand for output, and so forth have on investment, and what are influeuces of the difference in the degree of risk aversion on investment. K. R. Smith [21], [22], M. Rothschild and J. E. Stiglitz [18], [19], D. A. Diamond and J. E. Stiglitz [4], R. Hartman [11], [12], S. J. Nickell [16] and so on give interesting results about these problems. In this paper, however, we are rather mainly concerned with examining fixed investment behavior under uncertain demand and reinterpretating theories of investment, especially Acceleration Principle of Harrod type, Profit Principle of Kalecki–Kaldor type and Keynes' theoty of investment from a synthetic point of view based on our analysis. In the process we shall present a definition of the increase in demand uncertainty, by which we are able to show that the increased uncertainty in demand necessarily depresses investment.

A choice-theoretic interpretation of income–investment accelerator is given by H. I. Grossman [5]. It shows that an income–investment accelerator is obtained from the firm's optimizing behavior in the case where there appears excess supply in the output market, so that the firm faces the demand-restricted situation. In this paper we shall present another interpretation of Acceleration Principle by an optimizing behavior of the firm. We shall make clear that it will be possible for Acceleration Principle to be valid not only under excess supply of output but also under excess demand for output.

Our analysis will proceed as follows. In section 2 we shall investigate the state of short-term expectation and that of long-term expectation as a provision for analyzing fixed investment behavior. In section 3 we shall examine how the firm determines a level of investment under a putty-clay production function and a state of long-term expectation

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by maximizing expected utility of profits obtained from the investment. Section 4 will give a reinterpretation of both Acceleration Principle and Profit Principle from our point of view developed in section 3. Finally, in section 5, we shall discuss Keynes' theory of investment, especially the reason why the increase in investment brings about the diminishing marginal efficiency of investment.

2 Short-term and Long-term Expectations of the Firm

On deciding a level of price or output, the firm will presuppose the state of subjective expectation on the basis of which it estimates what will be obtained by the decision under the existing plant. By this state of expectation we may define *the state of short-term expectation of the firm*. When the firm plans some amount of fixed investment, however, it will take into consideration the prospective yields of the investment which will depend on the states of demand for output produced by the investment, the rate of price inflation and the rate of wage inflation from time to time during the life of the investment. The state of the firm's subjective expectation of them may we define as being *the state of long-term expectation of the firm*[1).]

Suppose the firm is either a price-setter at least in its short-run planning, the demand for whose output is uncertain but depends on the price level decided by it, or a price-taker, who faces uncertain demand independent from a given price. If the firm behaves as a price-taker, it cannot control the future demand in the short-run as well as in the long-run. And its future demand is a random variable not depending on price not only in the state of short-term expectation but also in the state of long-term expectation.

Provided that it is the price-setter, on the other hand, it could control the amount of demand to some extent by means of price manipulation at least in the short-run, even if demand is uncertain. But the longer the firm's planning period, the more events not anticipated at the present would occur and give great influences upon demand, so that the degree of controllability of demand should be weakened. Therefore, even though the firm is the price-setter, it would suppose that the future demand in the state of long-term expectation is a random variable which is independent from price. If this is true, it means that not only demand but also price are not controllable in the firm's planning of fixed investment. Accordingly, in addition to demand the firm should suppose a level (or levels) of price during the life of investment, which will be a kind of random variable depending upon various conditions surrounding the firm. It seems, however, that long-term demand is more stochastic than long-term price. Therefore we assume that on planning fixed investment the firm anticipates a definite level of future price on the one hand, and supposes demand as a random variable which is independent from the price on the other.

Thus, under our assumption, whether the firm is a price-taker or a price-setter, in the state of long-term expectation of the firm future demand is a random variable which does not depend on price, and price is an anticipated but definite variable.

3 Determination of Investment

Let us assume that the firm uses capital and labor to produce output through a produc-

1) Our definitions of the state of short-term expectation and that of long-term expectation correspond to Keynes' ones. See Keynes [15], pp. 147~148.

tion function, where the marginal productivity of capital or labor is positive and diminishing. We assume furthermore the production function is (1) quasi-concave with respect to capital and labor, (2) linear homogeneous of capital and labor, and (3) of putty-clay type (*i. e.*, capital and labor are *ex-ante* substitutable, but *ex-post* non-substitutable).

Suppose the rate of money-wages, w, and price of capital goods including interest payments during the life of plant, p_K, are given to the firm. Then from cost minimizing behavior to produce a given output under the production function we assumed, we can derive that optimal capital-output ratio, k, and labor-output ratio, n, are constant, respectively. They will change only when the relative price (w/p_K) varies, if there is no technical progress.

On planning fixed investment, the firm will consider demand for output during the life of the investment, part of which should be met by its present capacity, while the remaining part should be satisfied with product obtained from the investment. Denote the total future demand by Q, the present capacity output by Y, and $(Q-Y)$ by q. In the following, for simplicity, we shall term q demand. q is supposed to be a random variable which could be positive or negative, having a finite mean and variance.

Now suppose that the firm is examining how much it invests to satisfy demand q. We assume that the firm cannot sell its existing equipments to reduce the present stock of capital. Under this assumption, if the firm chooses a non-positive value of q, then there is no investment at all, *i. e.*, investment is zero. Therefore let us suppose here that the firm selects a given positive value of q, $\hat{q}$. Then there are three possible cases. At first it is possible that actual demand in the future is greater than or equal to $\hat{q}$. After choosing investing $\hat{I}=k\hat{q}$, maximum output to satisfy the future demand is equal to $\hat{q}$ because of *ex-post* non-substitutability between investment and labor. Therefore the firm's profits obtained from the investment in this case will be $(p-wn-p_K k)\hat{q}$, where p is an anticipated price of output.

Secondly, if future demand q is less than $\hat{q}$ but greater than zero, the firm will produce output by q by operating capital of kq with labor input nq, leaving the remaining capital $k(\hat{q}-q)$ as redundant, since capital and labor are *ex-post* non-substitutable. But capital cost in this case is not $p_K kq$ but $p_K k\hat{q}$. Thus the firm's profits will be $(pq-wnq-p_K k\hat{q})$ [2].

Thirdly, future demand could be equal to or less than zero. Then the firm will stop to operate capital $\hat{I}$ [3]. But the firm will suffer from capital cost of $p_K k\hat{q}$. Therefore its profits will be $(-p_K k\hat{q})$.

Let us suppose that the firm decides investment so as to maximize expected utility of profits obtained from it[4]. Denote profits by π and utility by u. The firm's utility function is,

2) Provided that output price, the rate of money-wages and the price of capital goods are given, that production function is linear homogeneous, and that the firm can sell output as it likes, the firm's profits will become equal to zero when it seeks to maximize profits. However, if demand for output is restricted to a given magnitude, it can maximize profits by minimizing cost to produce the given output. In this case the firm's profits are not always equal to zero, because a given price of output could be greater than marginal cost, which is equal to $(wn+p_K k)$.

3) When future demand is less than zero, the firm could operate $\hat{I}$ but remain part of the existing capital stock K as unused. For simplicity, however, we assume that this does not occur.

4) We neglect adjustment costs for investment in the following. In addition we do not take into consideration here the constraint on investment imposed by uncertainty of whether the firm is able to

we assume, given by

(3. 1) $$u=u(\pi),\ u'>0,\ u''\gtreqless 0.$$

That is, the firm could be either a risk-averter or the risk-neutral or a risk-taker. Denote the probability density function of q by $f(q)$. Then expected utility of profits of our firm will be given by

(3. 2) $$\begin{aligned} E[u(\pi)] &= u[(p-wn-p_K k)\hat{q}]\int_{\hat{q}}^{\infty} f(q)\,dq \\ &\quad + \int_0^{\hat{q}} u[(pq-wnq-p_K k\hat{q})]\cdot f(q)\,dq \\ &\quad + u[-p_K k\hat{q}]\int_{-\infty}^{0} f(q)\,dq. \end{aligned}$$

Suppose there is an interior maximum. Differentiating $E[u(\pi)]$ with respect to $\hat{q}$, we obtain the first order condition for a maximum of $E[u(\pi)]$

(3. 3) $$\begin{aligned} dE[u(\pi)]/d\hat{q} &= (p-wn-p_K k)\cdot u'[\pi(\hat{q},\hat{q})]\cdot r \\ &\quad - p_K k\int_0^{\hat{q}} u'[\pi(q,\hat{q})]\cdot f(q)\,dq - p_K k\cdot u'[\pi(0,\hat{q})]\cdot s = 0, \end{aligned}$$

where

(3. 4) $$r\equiv \int_{\hat{q}}^{\infty} f(q)\,dq,$$

(3. 5) $$s\equiv \int_{-\infty}^{0} f(q)\,dq,$$

and

(3. 6) $$\pi(q,\hat{q})\equiv pq-wnq-p_K k\hat{q},\ \text{for}\ 0\leqq q\leqq \hat{q}.$$

The second order condition is given by

(3. 7) $$\begin{aligned} d^2E[u(\pi)]/d\hat{q}^2 &= (p-wn-p_K k)^2\cdot u''[\pi(\hat{q},\hat{q})]\cdot r \\ &\quad - (p-wn)\cdot u'[\pi(\hat{q},\hat{q})]\cdot f(\hat{q}) \\ &\quad + (p_K k)^2\int_0^{\hat{q}} u''[\pi(q,\hat{q})]f(q)\,dq \\ &\quad + (p_K k)^2\cdot u''[\pi(0,\hat{q})]\cdot s<0. \end{aligned}$$

If the firm is either a risk-averter or the risk-neutral, equation (3.7) is satisfied. Even if it is a risk-taker, equation (3. 7) could be satisfied. We assume that if the firm is a risk-taker, its utility function satisfies equation (3. 7).

Rewriting equation (3. 3) we get

(3. 8) $$\theta=\mu/r$$

where we define

(3. 9) $$\theta\equiv (p-wn-p_K k)/p_K k,$$

and

(3. 10) $$\begin{aligned} \mu &\equiv \int_0^{\hat{q}} \{u'[\pi(q,\hat{q})]/u'[\pi(\hat{q},\hat{q})]\}\cdot f(q)\,dq \\ &\quad + \{u'[\pi(0,\hat{q})]/u'[\pi(\hat{q},\hat{q})]\}\cdot s,\ \text{for}\ 0\leqq q\leqq \hat{q}. \end{aligned}$$

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obtain loan for investment. We investigated the latter effect on investment in Fujino [7], pp. 58–74.

If the firm is risk-neutral, then $u'[\pi(q, \hat{q})]/u'[\pi(\hat{q}, \hat{q})]$ is always equal to unity, and equation (3. 8) reduces to

(3. 11) $$\theta = (1-r)/r.$$

When the firm is a risk-averter,

(3. 12) $$u'[\pi(q, \hat{q})]/u'[\pi(\hat{q}, \hat{q})] > 1, \text{ for } q < \hat{q}.$$

Therefore μ is greater than $(1-r)$. On the other hand, if the firm is a risk-taker,

(3. 13) $$u'[\pi(q, \hat{q})]/u'[\pi(\hat{q}, \hat{q})] < 1, \text{ for } q < \hat{q},$$

and μ should be less than $(1-r)$.

Differentiating (μ/r) with respect to $\hat{q}$, and taking into account the first order condition for a maximum (3. 8), we obtain

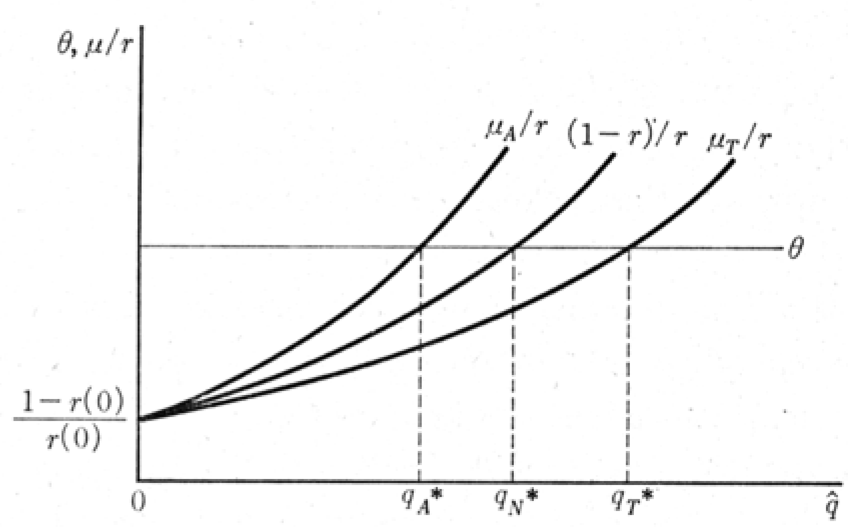
(3. 14) $$d(\mu/r)/d\hat{q} = (p_K k/r) \cdot [R_A(\hat{q}, \hat{q}) \cdot \theta^2 r + [(p-wn)/(p_K k)^2] \cdot f(\hat{q}) + \int_0^{\hat{q}} R_A(q, \hat{q}) \cdot \{u'[\pi(q, \hat{q})]/u'[\pi(\hat{q}, \hat{q})]\} \cdot f(q)\, dq + R_A(0, \hat{q}) \cdot \{u'[\pi(0, \hat{q})]/u'[\pi(\hat{q}, \hat{q})]\} \cdot s],$$

where

(3. 15) $$R_A(q, \hat{q}) \equiv -u''[\pi(q, \hat{q})]/u'[\pi(q, \hat{q})], \text{ for } 0 \leqq q \leqq \hat{q}.$$

That is, $R_A(q, \hat{q})$ is Arrow–Pratt absolute risk aversion[5]. We can easily show that $d(\mu/r)/d\hat{q}$ in equation (3. 14) should be positive, because of the second order condition for a maximum (3. 7). Thus (μ/r) for our firm is an increasing function of $\hat{q}$, being equal to $[(1-r(0))/r(0)]$ at $\hat{q}=0$, where $r(0)$ indicates the value of r at $\hat{q}=0$. And we obtain Fig. 1, where μ_A

Fig. 1



indicates μ in the case of risk-averter, and μ_T is μ in the case of risk-taker.

In order to make clear the economic meaning of (μ/r), let us introduce the rate of interest explicitly into our analysis. The price of capital goods, p_K, in the above includes not only costs to purchase capital goods invested but also interest payments per unit of capital goods invested during the duration period of investment. Denote the rate of interest over the whole duration period of capital goods by i, and the present price of capital goods by $\bar{p}_K$. Then

(3. 16) $$p_K = \bar{p}_K(1+i).$$

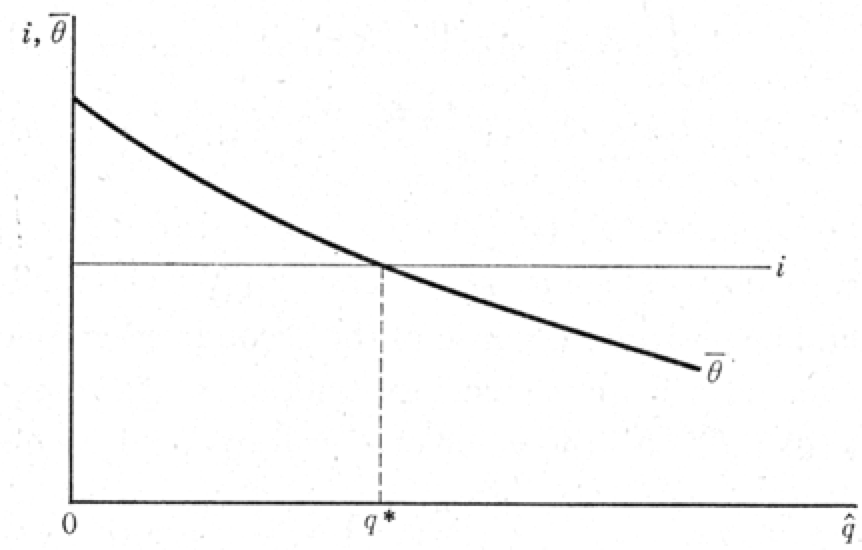
Substitution of equation (3. 16) into equation (3. 8) yields

(3. 17) $$[\{1/(1+(\mu/r))\} \cdot (p-wn) - \bar{p}_K k]/\bar{p}_K k = i.$$

5) K. Arrow [1], p. 33, G. W. Pratt [17].

Let us denote the left-hand side of equation (3. 17) by $\bar{\theta}$. From equation (3. 17) it is evident that optimal investment is determined at such a level that $\bar{\theta}$ becomes equal to the rate of interest, and that (μ/r) is nothing but the discount rate of risk with respect to gross profits brought about by investment. Thus, while θ is the expected rate of profits, where gross profits is discounted by the rate of interest because θ is equal to $[\{1/(1+i)\}\cdot(p-wn)-\bar{p}_K k]/\bar{p}_K k$, $\bar{\theta}$ is the one where gross profits is discounted by the rate of discount of risk. In what follows, we shall call θ *the externally expected rate of profits* and $\bar{\theta}$ *the internally expected rate of profits*. $\bar{\theta}$ is a decreasing function of $\hat{q}$, because (μ/r) will increase as $\hat{q}$ is increased. Therefore we obtain Fig. 2. If $\hat{q}=\infty$, r equals zero and μ takes a positive value, so that $\bar{\theta}$ should be equal to (-1). Therefore $\bar{\theta}$ curve has to cut the horizontal axis at a positive value of $\hat{q}$.

Fig. 2



It is evident from equation (3. 14) that the greater absolute risk aversion for a given $\hat{q}$, the greater becomes $d(\mu/r)/d\hat{q}$. Therefore (μ/r) curve for the firm having greater absolute risk aversion will shift upwards than otherwise. Thus we obtain

[*Proposition* 1]

Optimal investment will be determined at a level where the firm's externally expected rate of profits θ *is equal to the rate of disocunt of risk* (μ/r), *or the firm's internally expected rate of profirs* $\bar{\theta}$ *becomes equal to the rate of interest* i, *where the greater absolute risk aversion, the greater the rate of discount of risk* (μ/r) *and the less will be optimal investment. And optimal investment will be given by*

$$I^* = k(w/p_K)\cdot q^*, \tag{3. 18}$$

where q^* *is such a value of* $\hat{q}$ *that satisfies equation* (3. 8).

Suppose that the probability density function $f(q)$ of q shifts so that $r(q^*)$ at the previously optimal output q^* is diminished. Then new optimal output will fall compared with q^*, because (μ/r) curve will shift upwards under the assumed condition. Therefore, if we define the increase in uncertainty by such a shift of probability density function $f(q)$ that $r(q^*)$ is decreased, which means that the probability of fully operating the previously optimal level of q is now diminished, bringing about the increase in the rate of discount of risk (μ/r) at q^*, the effect of the increased uncertainty on investment should be negative. Therefore we get

[*Proposition* 2]

If uncertainty in demand is increased in the sense that $r(q^*)$ *is decreased, then the rate of* *discount of risk* (μ/r) *at* q^* *is so increased that investment will be diminished.*

Businessmen frequently say that "companies find uncertainty about the future as the most important single discouragement to investment[6]". If their meaning of the increase in uncertainty just matches that defined by economists, say, the mean preserving spread of a concerned random variable, then analyses by means of the latter could explain choice-theoretically actual behavior of investment. If it be not, however, there would be some distance between the actual behavior of investment and economists' analyses. The increase in uncertainty with respest to a random variable will have various aspects, some ones of which could be investigated by, say, the mean preserving spread, but the other would be not necessarily so. As far as we are concerned with the sluggish behavior of investment under the increased uncertainty told by businessmen, it seems that there exists some gap between the meaning of the increased uncertainty supposed by businessmen and the mean preserving spread. We hope, if possible, that our definition is rather near to the businessmen's one. And so far as we adopt our definition, the increase in uncertainty about future demand necessarily diminishes the level of investment.

4 Acceleration Principle and Profit Principle—A Synthesis—

The above analysis shows that there are at least three factors governing the size of investment. The first is the future demand q as a random variable subjectively perceived by the firm, whose shift will change (μ/r) to influence investment. The second is the firm's attitude towards risk, especially its absolute risk aversion, on which μ will depend. And the third is the externally expected rate of profits of the firm, θ, the upward shift of which will bring about an increase in investment. As shown in equation (3. 18), if q^* is increased owing to changes in these factors, investment I^* will be proportionately enlarged under a given (w/p_K)[7]. Therefore we obtain such an investment function as supposed by *Acceleration Principle*[8]. However the investment function does not mean that investment has a definite relationship to the increase in actual output, but means that it will proportionately vary with changes in the perceived demand, q^*, the size of which is determined by the above three factors, especially by the autonomously given, random variable q. Investment function due to Acceleration Principle has nothing to do with the optimizing behavior of firms, being strongly mechanical. But our investment function is reduced from an optimizing behavior of the firm under uncertain demand and will give an reinterpretation as well as a choice-theoretic basis to Acceleration Principle.

H. I. Grossman [5] develops a choice-theoretic interpretation of Acceleration Principle by the firm's maximizing behavior of the present value of net cash flow under excess supply of output, where the firm is restricted by the present sales of output because of the excess sup-

6) This sentence is quoted again from S. J. Nickell's quotation from Keith Richardson's comments on the N. E. D. O. report "Finance and Investment" (*Sunday Times* Business News, 1 June 1975). See S. J. Nickell [16], p. 47.

7) The real rate of wages (w/p_K) in terms of capital goods will be raised through time. Then, other things being equal, k will be increased. However, if labor augmenting technical progress occurs under competitive conditions, the real rate of wages with respect to labor in terms of efficiency unit would remain constant. We believe that k would keep a stable value through time for this reason.

8) See J. M. Clark [3], and R. F. Harrod [9], pp. 53~65; [10].

ply. It is an important contribution to our present problem. It seems, however, that it is, in a sense, a special case of our interpretation. He says that "profit maximizing behavior implies an income accelerator as part of the theoretical specification of investment demand, when, but only when output markets are depressed, that is to say characterized by excess supply." If we paraphrase his assumption in our context, we might say that he assumes that under excess supply of output the firm expects the present level of sales with a probability of unity as the future level of demand. This is a spectial case of our assumption that future demand is uncertain. And even though there is excess demand in the output market, the firm would not expect that it is able to sell its output so much as it likes, say infinitely, rather it would expect a limited amount of sales or a random amount with a finite mean and variance. Then Acceleration Principle could have a choice-theoretic basis for it not only when output markets are characterized by excess supply but also when they are in excess demand.

Our analysis, however, is closely connected with the investment function of Kalecki-Kaldor type based on the so-called *Profit Principle*, which asserts that investment depends positively not on the increase in output but on the absolute level of output, and negatively on the existing level of capital, on the other hand[9]. The future demand q is defined by the difference between the total future demand Q and the present capacity output Y. Under our assumption of *ex-post* non-substituability of capital and labor, we may suppose that the present stock of capital has a definite relation, $\bar{k}$, which is a function of the past series of (w/p_K), to the current capacity output, Y. Then we obtain from equation (3. 18)

$$I^* = k \cdot Q^* - (k/\bar{k})K, \tag{4. 1}$$

where Q^* indicates $(q^* + Y)$. Equation (4. 1) is nothing but Kalecki–Kaldor investment function. But, as in the case of income accelerator, investment depends not on the current level of output (or demand) but on a level of future demand perceived by the firm on the basis of maximizing expected utility of profits under an exogenously given, random demand Q. Because q^* depends on the expected rate of profits, Q^* also depends on the latter. Thus we may say that our investment function synthesizes both Acceleration Principle of Harrod type and Profit Principle of Kalecki-Kaldor type.

5 Uncertainty and the Marginal Efficiency of Investment

Now, let us examine Keynes' theory of investment from our theoretical point of view. As well known, according to his theory of investment the marginal efficiency of capital (or investment) decreases as investment is increased, and investment will be determined at such a level that the marginal efficiency of capital (or investment) will be equal to the rate of interest[10]. What matters in this case is the reasons why the marginal efficiency of capital will diminish as investment is increased. He says

> "If there is an increased investment in any given type of capital during any period of time, the marginal efficiency of that type of capital will diminish as the investment in it is increased, partly because the prospective yield will fall as the supply of that type of capital

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9) M. Kalecki [14], pp. 128∼141, and N. Kaldor [13], pp. 78∼92, *esp*. footnote 4).

10) T. Haavelmo [8], p. 172 asserts that if there is any relation between investment and the rate of interest, it must be the rate of change in the rate of interest rather than the interest-rate itself that matters. A. Sandmo [20] makes clear that investment in the short-run will be a decreasing function of the rate of interest, although the long-run relationship between the rate of interest and investment is ambiguous. Our analysis is concerned with the short-run effect in the context of Sandmo's analysis.

is increased, and partly because, as a rule, pressure on the facilities for producing that type of capital will cause its supply price to increase; the second of these factors being usually the more important in producing equilibrium in the short run, but the larger the period in view the more does the first factor take it place" (Keynes [15], p.136).

That is, he supposes that, as investment is increased, the decrease in the marginal efficiency of investment will be caused by both the decrease in the prospective yield of investment and the rise in the supply price of capital goods. If firms are the price-taker with respect to capital goods, we can neglect the second reason. And we are concerned with the first reason.

Then, what is the reason for the diminishing prospective yields of investment? Keynes does not give a clear reasoning about this question. We are recalled here that Keynes describes that his concept of marginal efficiency of capital has the same meaning as I. Fisher's *rate of return over cost* (Keynes [15], p. 140), and that Fisher asserts (investment) "opportunity is weakened because of the diminishing returns" (Fisher [6], p. 177). If we follow Fisher's interpretation, we can explain the diminishing marginal efficiency of investment by the decreasing marginal productivity of capital during the life of the investment under the assumption that the firm is a price-taker under a perfectly competitive product market, so that it supposes that it will be able to sell its output at an anticipated price as it likes. This interpretation based on the diminishing returns is linked to the line of thought of neo-classical theory[11)].

As pointed above, Keynes himself does not show the reason for the diminishing prospective yields of investment. His explanation is vague. But, from his theoretical view-point as a whole in which he is concerned with the demand-restricted economy where demand is uncertain, it is not consistent with his theory to suppose that firms can sell their output as they like at a price, even though they behave as the price-taker.

Therefore on interpretating the diminishing marginal efficiency of investment we should suppose that firms will examine at first how much output they will be able to sell in the future, that they will calculate next how much investment they need to produce the salable output, and that they will investigate thirdly how much prospective yields they will expect to obtain from the investment during its life. These suppositions seem to be suitable to understand the behavior of investment in the uncertain demand-resticted situation. And the diminishing marginal efficiency of investment would be due to the decrease in probability for sales of output as investment, therefore, output is increased, rather than the diminishing marginal productivity of capital. Let us clarify this point in terms of our analysis.

In order to make clear the correspondence of our analysis to Keynes' theory, let us follow two period analysis. Then we may define the marginal efficiency of investment ρ by the following equation;

(5. 1) $$\bar{p}_K = R/(1+\rho),$$

Where R is the prospective yield of investment per unit of capital goods. Therefore ρ is expressed by

(5. 2) $$\rho = (R-\bar{p}_K)/\bar{p}_K = (Rk-\bar{p}_K k)/\bar{p}_K k.$$

Rk expresses the prospective yield of investment per unit of output, corresponding to $\{1/(1+(\mu/r)\}\cdot(p-wn)$ in $\bar{\theta}$ in equation (3. 17). Comparing the left-hand side of equation (3. 17), which expresses $\bar{\theta}$, with ρ defined by equation (5. 2), it is evident that the internally

11) We have ever adapted this kind of interpretation of the declining marginal efficiency of investment (Fujino [7], ch. 9). However, now we are inclined to take rather another explanation as shown later.

expected rate of profits $\bar{\theta}$ is nothing less than the marginal efficiency of investment.

On analyzing the behavior of investment, Keynes distinguishes two types of risk. He writes

> "The first is the entrepreneur's or borrower's risk and arises out of doubts in his own mind as to the probability of his actually earning the prospective yield for which he hopes.…… But where a system of borrowing and lending exists,…… a second type of risk is relevant which we may call the lender's risk.…… The second, however, is a pure addition to the cost of investment which would not exist if the borrower and lender were the same person(Keynes[15],p.144).

The first type of risk concerns us here. In terms of our analysis it is expressed by the discount rate of risk (μ/r) in $\bar{\theta}$. The more sales does the firm plan, and the more risk averse is it, the greater the rate of discount (μ/r).

Now, it is evident that the marginal efficiency of investment $\bar{\theta}$ in terms of our analysis will be diminished as investment is increased, not because the marginal productivity of capital is decreased, but because the discount rate of risk (μ/r) is increased, *i. e.*, because the first type of risk referred by Keynes is increased[12].

Finally let us investigate briefly the phenomenon that investment is not sensitive to the change in the rate of interest. Suppose that, as $\hat{q}$ increases, $r(\hat{q})$, which is such a probability that future demand greater than or equal to $\hat{q}$ will appear, is suddenly diminished and correspondingly $\mu(\hat{q})$ is increased abruptly at a certain value of $\hat{q}$. Then $\bar{\theta}$ curve would be nearly vertical to $\hat{q}$ axis at that magnitude of $\hat{q}$, so that there would be scarcely any increase in investment even though the rate of interest falls, so long as the rate of interest changes within the range of this vertical part of $\bar{\theta}$ curve. If the firm's probabilistic judgement about future demand is of this type, then to operate the rate of interest would be not effective in order to manipulate effective demand.

6 Conclusions

In this paper we assumed that the firm determines a level of investment so as to maximize expected utility of profits obtained from the investment under a linear homogeneous, quasi-concave, putty-clay production function and a state of long-term expectation where future demand is uncertain and future output price is an expected, but definite value. And we clarified the following points.

1. An optimal level of investment will have a fixed relationship, which depends on the relative price between the rate of wages and the price of capital goods including interest payments, with a level of future demand, which is defined by the difference between a level of total future demand and the present capacity output. The level of future demand will be so determined that the firm's externally (or internally) expected rate of profits may become equal to the discount rate of risk with respect to gross profits (or the rate of interest), where the discount rate of risk depends on the firm's attitude towards risk as well as its subjective probability density function of future demand. And the increase in uncertainty about the future demand, in the sense that the probability of realization of demand greater than or equal to the previous level of optimal output is diminished, which is expressed by the rise in the discount rate of risk, will decrease the level of investment.

12) The reason why $\bar{\theta}$ is not diminished due to the diminishing returns lies in what follows; we supposes that production function is linear homogeneous, so that the capital-labor ratio is fixed, therefore the marginal productivity of capital is constant, so far as the relative price (w/p_K) remains constant.

2. Acceleration Principle could have a choice-theoretic basis not only under excess supply of output but also under excess demend for output. At the same time, Profit Principle of Kalecki–Kaldor type could be closely connected to Acceleration Principle, being able to be explained by our approach to investment behavior under uncertain demand.

3. The diminishing tendency of the marginal efficiency of investment due to the increase in investment, which Keynes asserts, would be explained by the increasing risk the firm feels about greater magnitude of future demand (which is expressed by the increase in the rate of discount of risk).

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