An Effect of Updating Base Period on the Rate of Change in Price Index*

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1. Introduction

Numerous methods of weighting in price index numbers have been developed and analyzed in the literature. For the most part this is done in terms of levels of various price indexes. But it seems that consumers and the general public are more concerned with the rates of change of price indexes than with their levels. Accordingly, in this note we investigate an aspect of the rate of change of price indexes, that is, an effect of updating a base period on the rate of change in price indexes.

Some consumer groups claim that whenever the government updates the base year of the price index, the rate of increase in the Laspeyres price index tends to fall compared with that of the previous base year. As a clue we examine the conditions under which updating the base period causes the rate of increase in price index to rise or fall. We show two versions of these conditions: a strong one and a weak one. This is analyzed only in terms of a statistical (atomistic) approach, but may possibly have some implications in terms of an economic (functional) approach which are not developed here[1].

Suppose $P_\alpha(s, t)$ denotes a price index in period t with period s as the base, where subscript α is a form of the price index ($s \leqq t$). In this note the base period means the period as both a reference base and a weighting base. Then, the Laspeyres and Paasche price indexes are

$$P_L(s,t) \equiv \sum_{i=1}^{n} p_t^i q_s^i \Big/ \sum_{i=1}^{n} p_s^i q_s^i,$$

$$P_P(s,t) \equiv \sum_{i=1}^{n} p_t^i q_t^i \Big/ \sum_{i=1}^{n} p_s^i q_t^i,$$

where n is the number of commodities, and p_τ^i and q_τ^i are, respectively, the price and the quantity of the i-th commodity in period τ.

Here, it is assumed that

$$p_\tau^i \geqq 0, \quad q_\tau^i \geqq 0 \text{ for any } i \text{ and } \tau,$$

$$\sum_{i=1}^{n} p_\sigma^i q_\tau^i > 0 \text{ for any } \sigma \text{ and } \tau.$$

Next, suppose $\delta_\alpha(s, t; \sigma)$ denotes the rate of change in α-form price index between period s and period t with period σ as the base ($\sigma \leqq s < t$). Then,

$$\delta_\alpha(s,t;\sigma) \equiv P_\alpha(\sigma,t)/P_\alpha(\sigma,s) - 1.$$

2. Strong Conditions

At first, we show a set of strong sufficient conditions under which updating the base period leads to an increase or a decrease in the rate of change of the Laspeyres price index. In this section prices and quantities of all commodities at any period are assumed to be positive. If the base period is updated from period σ to period τ, then the change in the rate of increase of the Laspeyres price index between periods s and t is for $\sigma < \tau \leqq s < t$

$$\begin{aligned}\delta_1 &\equiv \delta_L(s,t;\tau) - \delta_L(s,t;\sigma) \\ &= P_L(\tau,t)/P_L(\tau,s) - P_L(\sigma,t)/P_L(\sigma,s) \\ &= \sum p_t q_\tau / \sum p_s q_\tau - \sum p_t q_\sigma / \sum p_s q_\sigma. \end{aligned} \qquad (1)$$

Then, we get

$$\delta_1 = \sum_{i=1}^{n} \sum_{j=i+1}^{n} (p_s^i p_t^j - p_t^i p_s^j)(q_\sigma^i q_\tau^j - q_\tau^i q_\sigma^j) \Big/ \left(\sum_{i=1}^{n} p_s^i q_\tau^i \sum_{i=1}^{n} p_s^i q_\sigma^i \right)^{2)}. \qquad (2)$$

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1) On these approaches see, for example, Frisch [4], Tokoyama [7].

2) The proof is the following.

$$\delta_1 = [1/(\sum p_s q_\tau \sum p_s q_\sigma)](\sum p_t q_\tau \sum p_s q_\sigma - \sum p_t q_\sigma \sum p_s q_\tau).$$

Defining $F_1 \equiv 1/(\sum p_s q_\tau \sum p_s q_\sigma)$, then $F_1 > 0$ and

$$\begin{aligned}\delta_1 &= F_1 \left(\sum_{i=1}^{n} \sum_{j=1}^{n} p_t^i q_\tau^i p_s^j q_\sigma^j - \sum_{i=1}^{n} \sum_{j=1}^{n} p_t^i q_\sigma^i p_s^j q_\tau^j \right) \\ &= F_1 \sum_{i=1}^{n} \left[\sum_{j \neq i} p_t^i p_s^j q_\tau^i q_\sigma^j + p_t^i p_s^i q_\tau^i q_\sigma^i - \left(\sum_{j \neq i} p_t^i p_s^j q_\sigma^i q_\tau^j + p_t^i p_s^i q_\sigma^i q_\tau^i \right) \right] \\ &= F_1 \sum_{i=1}^{n} \sum_{j \neq i} p_t^i p_s^j (q_\tau^i q_\sigma^j - q_\sigma^i q_\tau^j) \\ &= F_1 \left[\sum_{i=1}^{n} \sum_{j=1}^{i-1} p_t^i p_s^j (q_\tau^i q_\sigma^j - q_\sigma^i q_\tau^j) \right. \end{aligned}$$

We now introduce several conditions which are used in the propositions below.

C. 1 For any i and j, if $p_\tau^i/p_\sigma^i \leqq p_\tau^j/p_\sigma^j$, then $q_\tau^i/q_\sigma^i \geqq q_\tau^j/q_\sigma^j$.

C. 1′ For any i and j, if $p_t^i/p_s^i \leqq p_t^j/p_s^j$, then $q_t^i/q_s^i \geqq q_t^j/q_s^j$.

C. 2 For any i and j, if $p_\tau^i/p_\sigma^i \leqq p_\tau^j/p_\sigma^j$, then $p_t^i/p_s^i \leqq p_t^j/p_s^j$.

C. 2′ For any i and j, if $p_\tau^i/p_\sigma^i \leqq p_\tau^j/p_\sigma^j$, then $p_t^i/p_s^i \geqq p_t^j/p_s^j$.

Proposition 1

If conditions C. 1 and C. 2 are satisfied[3)], then

$$\delta_L(s, t\,;\tau) - \delta_L(s, t\,;\sigma) \leqq 0.$$

More specifically, if the necessary conditions of C. 1 and C.2 hold in inequality for a pair i and j, then

$$\delta_L(s, t\,;\tau) - \delta_L(s, t\,;\sigma) < 0.$$

On the other hand, if C. 1 and C. 2′ are satisfied[3′)], then

$$\delta_L(s, t\,;\tau) - \delta_L(s, t\,;\sigma) \geqq 0.$$

More specifically, if the necessary conditions of C. 1 and C. 2′ hold in inequality for a pair i and j, then

$$\delta_L(s, t\,;\tau) - \delta_L(s, t\,;\sigma) > 0.$$

Condition C. 1 (C. 1′) means that for any pair of commodities, if the price change $p_\tau/p_\sigma (p_t/p_s)$ of one commodity is larger than the price change of the other, then the quantity change $q_\tau/q_\sigma (q_t/q_s)$ of the first commodity will be smaller than that of the second during the same term. Condition C. 2 (C. 2′) means that for any pair of commodities, if the price change of one commodity is larger than that of the other between periods σ and τ, then the price change of the first commodity will be larger (smaller) than that of the second between periods s and t. Therefore, the economic meaning of this proposition is the following. For any two commodities, if the price change of one commodity is larger in the interval of updating the base period, its quantity change in that same interval will be smaller and its price change in the interval concerned with the rate of price change will be larger (smaller) in comparison to the second commodity; and if these conditions are satisfied, then updating the base period tends to lower (raise) the rate of increase in the Laspeyres price index.

Proof of Proposition 1

At first, we change and renumber the order of n commodities such that for any $i<j$ $p_\tau^i/p_\sigma^i \leqq p_\tau^j/p_\sigma^j$. Then, for any $i<j$, $q_\sigma^i q_\tau^j - q_\tau^i q_\sigma^j \leqq 0$ and $p_s^i p_t^j - p_t^i p_s^j \geqq 0$ from C. 1 and C. 2. Thus, for any $i<j$

$$(p_s^i p_t^j - p_t^i p_s^j)(q_\sigma^i q_\tau^j - q_\tau^i q_\sigma^j) \leqq 0.$$

Since the denominator is positive in (2), $\delta_1 \leqq 0$. If the necessary conditions of C. 1 and C. 2 hold in inequality for a pair, $i<j$, then for the pair

$$(p_s^i p_t^j - p_t^i p_s^j)(q_\sigma^i q_\tau^j - q_\tau^i q_\sigma^j) < 0.$$

Thus, from (2), $\delta_1 < 0$.

Similarly, we can derive the latter half of the proposition by C. 1 and C. 2′.

Next, we investigate the case of the Paasche price index. If the base period is updated from period σ to period τ, then the change in the rate of increase of the Paasche price index between periods s and t is for $\sigma<\tau\leqq s<t$

$$\begin{aligned}\delta_2 &\equiv \delta_P(s, t\,;\tau) - \delta_P(s, t\,;\sigma)\\ &= P_P(\tau, t)/P_P(\tau, s) - P_P(\sigma, t)/P_P(\sigma, s)\\ &= (\textstyle\sum p_t q_t/\sum p_s q_s)(\sum p_\tau q_s/\sum p_\tau q_t\\ &\quad -\textstyle\sum p_\sigma q_s/\sum p_\sigma q_t).\end{aligned} \tag{3}$$

Then, we get

$$\delta_2 = \left[\sum_{i=1}^n p_t^i q_t^i \Big/ \left(\sum_{i=1}^n p_\tau^i q_t^i \sum_{i=1}^n p_\sigma^i q_t^i \times \sum_{i=1}^n p_s^i q_s^i\right)\right]\left[\sum_{i=1}^n \sum_{j=i+1}^n (p_\tau^i p_\sigma^j - p_\sigma^i p_\tau^j) \times (q_s^i q_t^j - q_t^i q_s^j)\right]^{4)}. \tag{4}$$

$$\begin{aligned}&+\sum_{i=1}^n \sum_{j=i+1}^n p_t^i p_s^j (q_\tau^i q_\sigma^j - q_\sigma^i q_\tau^j)\Bigg]\\ &= F_1\Bigg[\sum_{j=1}^n \sum_{i=j+1}^n p_t^i p_s^j (q_\tau^i q_\sigma^j - q_\sigma^i q_\tau^j)\\ &\quad +\sum_{i=1}^n \sum_{j=i+1}^n p_t^i p_s^j (q_\tau^i q_\sigma^j - q_\sigma^i q_\tau^j)\Bigg]\\ &= F_1\Bigg[\sum_{i=1}^n \sum_{j=i+1}^n p_s^i p_t^j (q_\sigma^i q_\tau^j - q_\tau^i q_\sigma^j)\\ &\quad +\sum_{i=1}^n \sum_{j=i+1}^n p_t^i p_s^j (q_\tau^i q_\sigma^j - q_\sigma^i q_\tau^j)\Bigg]\\ &= F_1 \sum_{i=1}^n \sum_{j=i+1}^n (p_s^i p_t^j - p_t^i p_s^j)(q_\sigma^i q_\tau^j - q_\tau^i q_\sigma^j).\end{aligned}$$

3), 3′) However, when the sufficient conditions of C. 1 and C. 2 (C. 2′) hold in equality, we assume that C. 1 and C. 2 (C. 2′) are satisfied for the same pair i and j.

4) The proof is the following.

$$\delta_2 = [\textstyle\sum p_t q_t/(\sum p_\tau q_t \sum p_\sigma q_t \sum p_s q_s)]\,(\sum p_\tau q_s \sum p_\sigma q_t - \sum p_\sigma q_s \sum p_\tau q_t).$$

Defining $F_2 \equiv \sum p_t q_t/(\sum p_\tau q_t \sum p_\sigma q_t \sum p_s q_s)$, then $F_2 > 0$ and

$$\delta_2 = F_2\left(\sum_{i=1}^n \sum_{j=1}^n p_\tau^i q_s^i p_\sigma^j q_t^j - \sum_{i=1}^n \sum_{j=1}^n p_\sigma^i q_s^i p_\tau^j q_t^j\right)$$

Proposition 2

If conditions C. 1′ and C. 2 above are satisfied [5)], then

$\delta_P(s,t;\tau)-\delta_P(s,t;\sigma)\geqq 0.$

More specifically, if the necessary condition of C. 1′ and the sufficient condition of C. 2 hold in inequality for a pair i and j, then

$\delta_P(s,t;\tau)-\delta_P(s,t;\sigma)>0.$

On the other hand, if C. 1′ and C. 2′ are satisfied [5′)], then

$\delta_P(s,t;\tau)-\delta_P(s,t;\sigma)\leqq 0.$

More specifically, if the necessary condition of C. 1′ and the sufficient condition of C. 2′ hold in inequality for a pair i and j, then

$\delta_P(s,t;\tau)-\delta_P(s,t;\sigma)<0.$

Thus, if condition C. 2 (C. 2′) is satisfied—for any two commodities, if the price change of one commodity is larger in the interval of updating the base, its price change will be larger (smaller) in the interval concerned with the rate of price change in comparison to the second commodity; and if condition C. 1′ is satisfied—for any two commodities, if the price change of one commodity is larger in the interval concerned with the rate of price change, its quantity change will be smaller in the same interval in comparison to the second commodity; then updating the base tends to raise (lower) the rate of increase in the Paasche price index.

Proof of Proposition 2

First, we change and renumber the order of n commodities such that for any $i<j$, $p_\tau^i/p_\sigma^i \leqq p_\tau^j/p_\sigma^j$. Then, for any $i<j$, $p_\tau^i p_\sigma^j - p_\sigma^i p_\tau^j \leqq 0$, and from C. 2 and C. 1′ $q_t^i/q_s^i \geqq q_t^j/q_s^j$, that is, $q_s^i q_t^j - q_t^i q_s^j \leqq 0$. Thus,

$(p_\tau^i p_\sigma^j - p_\sigma^i p_\tau^j)(q_s^i q_t^j - q_t^i q_s^j)\geqq 0$ for any $i<j$.

Since the term in the first brackets is positive in equation (4),

$\delta_P(s,t;\tau)-\delta_P(s,t;\sigma)\geqq 0.$

If the necessary condition of C. 1′ and the sufficient condition of C. 2 hold in inequality for a pair of $i<j$, then for the pair

$(p_\tau^i p_\sigma^j - p_\sigma^i p_\tau^j)(q_s^i q_t^j - q_t^i q_s^j)>0.$

Thus, from (4)

$\delta_P(s,t;\tau)-\delta_P(s,t;\sigma)>0.$

Similarly, we can derive the latter half of the proposition from C. 1′ and C. 2′.

3. Weak Conditions

In this section we show a set of necessary and sufficient conditions under which updating the base period brings about an increase or a decrease in the rate of increase of the Laspeyres and Paasche price indexes.

Firstly, for later use we describe the following formula by Hisatsugu and Ide [5] which is an extension of that by Bortkiewicz [3]:

$$\sum uvw \sum w/(\sum uw \sum vw) = 1 + C(u,v;w)^{6)}, \tag{5}$$

where $C(u,v;w)\equiv \sum w(u/\bar{u}-1)(v/\bar{v}-1)/\sum w$,

$\bar{u}\equiv \sum uw/\sum w,\quad \bar{v}\equiv \sum vw/\sum w,$

$\sum w>0,\quad \sum uw>0,\quad \sum vw>0.$

Then, $C(u,v;w)$ is regarded as a variant of the weighted covariance between u and v with w as weights. Moreover, defining

$$\sigma_u^2 \equiv \sum w(u-\bar{u})^2/\sum w,\quad \sigma_v^2\equiv \sum w(v-\bar{v})^2/\sum w,\quad r_{uv}\equiv[\sum w(u-\bar{u})(v-\bar{v})/\sum w]/(\sigma_u\sigma_v),$$

$$\begin{aligned}
&= F_2 \sum_{i=1}^{n}\Big[\sum_{j\neq i} p_\tau^i p_\sigma^j q_s^i q_t^j + p_\tau^i p_\sigma^i q_s^i q_t^i \\
&\quad - \Big(\sum_{j\neq i} p_\sigma^i p_\tau^j q_s^i q_t^j + p_\sigma^i p_\tau^i q_s^i q_t^i\Big)\Big] \\
&= F_2 \sum_{i=1}^{n}\sum_{j\neq i} q_s^i q_t^j (p_\tau^i p_\sigma^j - p_\sigma^i p_\tau^j) \\
&= F_2 \Big[\sum_{i=1}^{n}\sum_{j=i}^{i-1} q_s^i q_t^j (p_\tau^i p_\sigma^j - p_\sigma^i p_\tau^j) \\
&\quad + \sum_{i=1}^{n}\sum_{j=i+1}^{n} q_s^i q_t^j (p_\tau^i p_\sigma^j - p_\sigma^i p_\tau^j)\Big] \\
&= F_2 \Big[\sum_{i=1}^{n}\sum_{j=i+1}^{n} q_t^i q_s^j (p_\sigma^i p_\tau^j - p_\tau^i p_\sigma^j) \\
&\quad + \sum_{i=1}^{n}\sum_{j=i+1}^{n} q_s^i q_t^j (p_\tau^i p_\sigma^j - p_\sigma^i p_\tau^j)\Big] \\
&= F_2 \sum_{i=1}^{n}\sum_{j=i+1}^{n} (p_\tau^i p_\sigma^j - p_\sigma^i p_\tau^j)(q_s^i q_t^j - q_t^i q_s^j).
\end{aligned}$$

5), 5′) However, when the sufficient condition of C. 1′ and the necessary condition of C. 2 (C. 2′) hold in equality, we assume that C. 1′ and C. 2 (C. 2′) are satisfied for the same pair i and j.

6) We restate their proof here.

$$\begin{aligned}
&\sum (u-\bar{u})(v-\bar{v})w \\
&= \sum uvw - \bar{u}\sum vw - \bar{v}\sum uw + \bar{u}\bar{v}\sum w \\
&= \sum uvw - \bar{u}\bar{v}\sum w - \bar{u}\bar{v}\sum w + \bar{u}\bar{v}\sum w \\
&= \sum uvw - \bar{u}\bar{v}\sum w.
\end{aligned}$$

Dividing both sides by $\bar{u}\bar{v}\sum w$ and rearranging gives

$$\sum uvw/(\bar{u}\bar{v}\sum w) = 1+\sum (u-\bar{u})(v-\bar{v})w/(\bar{u}\bar{v}\sum w).$$

Then,

$$\sum uvw/[(\sum uw/\sum w)(\sum vw/\sum w)\sum w] = 1+\sum (u/\bar{u}-1)(v/\bar{v}-1)w/\sum w.$$

Thus, (5) is obtained.

we obtain $C(u, v; w)=r_{uv}(\sigma_u/\bar{u})(\sigma_v/\bar{v})$. This means that $C(u, v; w)$ has the same sign as r_{uv}, the weighted correlation coefficient between u and v.

Next, we state briefly what has already been analyzed in the literature on the relations between the Laspeyres direct index and the Laspeyres chain index so as to use it in comparison with the result obtained later. The Laspeyres chain index in period t with period s as the base is

$$P_{LC}(s, t) \equiv P_L(s, s+1) \cdot P_L(s+1, s+2) \cdots P_L(t-1, t).$$

Defining $D_{rst} \equiv P_L(r, s) P_L(s, t) / P_L(r, t)$ for $r<s<t$,

$$P_{LC}(s, t) / P_L(s, t) = D_{s,s+1,s+2} \cdot D_{s,s+2,s+3} \cdots D_{s,t-1,t}. \quad (6)$$

Frisch [4], by using his formula in terms of correlation between price changes, claims that the chain index tends to drift upwards from the direct index. Then, Zarnowitz [8] derives for $s+2 \leqq k \leqq t$

$$D_{s,k-1,k}=1+C(p_k/p_{k-1}, q_{k-1}/q_s; p_{k-1}q_s)^{7)}. \quad (7)$$

Thus, if the correlation between the price change p_k/p_{k-1} and the quantity change q_{k-1}/q_s is positive (negative) for $s+2 \leqq k \leqq t$, the chain index drifts up (down) from the direct index. While Zarnowitz admits a tendency of the upward drift of the chain index when chaining is relatively short and when seasonal movements occur in prices and quantities, Allen [1, 2], Morita [6], Hisatsugu and Ide [5], and others show that there is no empirical evidence supporting the drift.

Now, we consider the conditions under which updating the base causes the rate of increase of the Laspeyres price index to increase or decrease. When the base period is updated from period σ to period τ, the change in the rate of increase of the Laspeyres price index between periods s and t is from (1)

$$\delta_L(s, t; \tau) - \delta_L(s, t; \sigma) = \sum p_t q_\tau / \sum p_s q_\tau - \sum p_t q_\sigma / \sum p_s q_\sigma \text{ for } \sigma<\tau \leqq s<t.$$

Since both terms in the right-hand side are positive, a set of necessary and sufficient conditions for $\delta_L(s, t; \tau) > (<) \delta_L(s, t; \sigma)$ is obtained by dividing the first term by the second. That is,

$$R_1 \equiv (\sum p_t q_\tau / \sum p_s q_\tau)(\sum p_s q_\sigma / \sum p_t q_\sigma) > (<) 1.$$

From (5),

$$R_1 = \frac{\sum[(p_t/p_s)(q_\tau/q_\sigma)p_s q_\sigma] \sum p_s q_\sigma}{\sum[(p_t/p_s)p_s q_\sigma] \sum[(q_\tau/q_\sigma)p_s q_\sigma]} = 1 + C(p_t/p_s, q_\tau/q_\sigma; p_s q_\sigma).$$

Thus, we obtain the following proposition.

Proposition 3

$\delta_L(s, t; \tau) > (<) \delta_L(s, t; \sigma)$ for $\sigma<\tau \leqq s<t$, if and only if

$C(p_t/p_s, q_\tau/q_\sigma; p_s q_\sigma) > (<) 0$.

That is, if and only if there is a positive (negative) correlation coefficient between the price change in the interval concerned with the rate of price change p_t/p_s and the quantity change in the interval of updating the base q_τ/q_σ, updating the base period from σ to τ tends to raise (lower) the rate of increase of the Laspeyres price index between periods s and t.

If $\sigma=s$, $\tau=k-1$, $s=k-1$, and $t=k$, then $R_1=1+C(p_k/p_{k-1}, q_{k-1}/q_s; p_{k-1}q_s)$. This is the term which is used in comparison between the Laspeyres chain index and the Laspeyres direct index, that is, $D_{s,k-1,k}$ in equation (7). Therefore, the necessary and sufficient condition for updating the base period to bring about raising (lowering) of the rate of increase of the Laspeyres price index is similar to the sufficient condition for the Laspeyres chain index to drift up (down) from the Laspeyres direct index. However, there is the difference that in the former only one correlation has to be met, whereas in the latter all the correlations have to be satisfied for $s+2 \leqq k \leqq t$.

Next, we turn to the case of the Paasche price index. When the base period is updated from period σ to period τ, the change in the rate of increase of the Paasche price index between periods s and t is, from (3),

$$\delta_P(s, t; \tau) - \delta_P(s, t; \sigma) = (\sum p_t q_t / \sum p_s q_s) \times (\sum p_\tau q_s / \sum p_\tau q_t - \sum p_\sigma q_s / \sum p_\sigma q_t) \text{ for } \sigma<\tau \leqq s<t.$$

Since all the terms in the right-hand side are positive, a set of necessary and sufficient condition for $\delta_P(s, t; \tau) > (<) \delta_P(s, t; \sigma)$ is obtained by dividing the first term by the second inside of the second parentheses. That is,

$$R_2 \equiv \sum p_\tau q_s \sum p_\sigma q_t / (\sum p_\tau q_t \sum p_\sigma q_s) > (<) 1.$$

From (5),

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7) It is derived as follows.

$$D_{s,k-1,k} = P_L(s, k-1) P_L(k-1, k) / P_L(s, k) = (\sum p_{k-1}q_s / \sum p_s q_s)(\sum p_k q_{k-1} / \sum p_{k-1}q_{k-1}) / (\sum p_k q_s / \sum p_s q_s) = \frac{\sum[(p_k/p_{k-1})(q_{k-1}/q_s)p_{k-1}q_s] \sum p_{k-1}q_s}{\sum[(p_k/p_{k-1})p_{k-1}q_s] \sum[(q_{k-1}/q_s)p_{k-1}q_s]}.$$

Then, (7) is obtained from (5).

$$R_2=\frac{\sum[(p_\tau/p_\sigma)p_\sigma q_s]\sum[(q_t/q_s)p_\sigma q_s]}{\sum[(p_\tau/p_\sigma)(q_t/q_s)p_\sigma q_s]\sum p_\sigma q_s}$$

$$=\frac{1}{1+C(p_\tau/p_\sigma, q_t/q_s; p_\sigma q_s)}.$$

Thus, we obtain the following proposition.

Proposition 4

$$\delta_P(s,t;\tau)-\delta_P(s,t;\sigma)>(<)0 \text{ for } \sigma<\tau\leqq s<t,$$

if and only if

$$C(p_\tau/p_\sigma, q_t/q_s; p_\sigma q_s)<(>)0.$$

That is, if and only if there is a negative (positive) correlation coefficient between the price change in the interval of updating the base p_τ/p_σ and the quantity change in the interval concerned with the rate of price change q_t/q_s, updating the base period from σ to τ tends to raise (lower) the rate of increase of the Paasche price index between periods s and t.

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