ON THE ENDOGENOUS SUPPLY OF MONEY*

AKIRA TAKAYAMA • JOHN Z. DRABICKI

It is now well-known that the usual practice in most macro textbooks which takes the money supply as an exogenous variable, although a useful theoretical device for simplification, often leads to erroneous results by blurring the role played by the commercial banks and the central bank. That the money supply is an endogenous variable should be apparent since it, for example, depends on the lending behavior of commercial banks. Thus, among others, Tobin writes that the assumption of an exogenous money supply is "wrong," and attributes it to "be a residue of the days when money was 100 percent government issue currency" ([37], p. 169)[1]. In fact, there has been a considerable amount of effort in the profession both theoretical and empirical, especially since the early 1960's, to "endogenize" the money supply[2].

Having studied these works, we elsewhere [13] proposed the formula which would play the role of the money supply equation, and then showed its application in the "money and growth" problem. The first purpose of the present paper is to show some other applications, which will be useful and illuminating, including its effect on the LM-curve. Secondly, we intend to survey briefly the literature on the endogenous supply of money in a unified fashion. This will be useful since the literature is very diversified and complicated, and since it will put our work [13] in a proper perspective. To make this paper sufficiently self-contained, we shall briefly summarize the Drabicki-Takayama [13] formula of the money supply in Section II. Sections III and IV are, respectively, concerned with some applications of this formula and a survey of the money supply literature.

I. Framework

We consider an economy consisting of four sectors, the government, the central bank, the (commercial) banks, and the public (the households and the firms), which deal with five types of assets: physical goods, the central bank's currency, demand deposits, the bank reserves and interest yielding securities ("bonds"). The securities consist of both private and government securities, which are assumed to be indistinguishable to the public in all respects such as the expected rate of return, risk, etc.

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1) Patinkin even states that since "neither the volume of bank credit nor that of demand deposits is exogenously determined," it is "meaningless to inquire about the effects of (say) an expansion in (demand deposits) ......upon the system" ([27], p. 300).

2) Harry Johnson in his famous survey on monetary theory [22] commented that money supply theory had been thoroughly neglected in monetary analysis. This is misleading, as Brunner points out, "during the 1920's until about middle thirties money supply analysis made substantial progress' ([6], p. 89). He then briefly mentions the works by A. Phillips, R. Burgess, W. Riefler, L. Currie, and others during this period. See also Meigs ([24], esp. chap. II). The development of money supply analysis "stagnated thoroughly after 1935" ([6], p. 91) until the 1960's.

The money supply consists of the central bank's currency and demand deposits. The omission of government money, vault cash, and time deposits is solely for the sake of simplicity and the reader should easily be able to incorporate them into the present analysis.

In Table 1 we summarize the symbols used and the balance sheet of each sector where all entries are measured in monetary units (cf. Tobin [38]). This table also summarizes some assumptions used in this paper. A positive entry represents an asset of a particular sector and a negative entry represents its liability. The symbols S^p, S^b, and S^c, respectively, represent the *net* holding (stock excess demand) of securities by the public, the commercial banks, and the central bank. The sum of a column for a particular sector represents the net worth of that sector, which means the balance sheet identity. The entry in the last column is the net exogenous supply of a particular asset. For stocks of goods (or "equities"), this is the economy's inheritance from the past, with price denoted by p. The government is a net debtor whose debt consists only of government bonds, and whose amount is equal to D ("dollars"). It is assumed for the sake of simplicity that the central bank's holding of securities (S^c) is an exogenous (or a policy) variable. In an open economy, a part of S^c corresponds to the foreign exchange reserves that the country possesses. Noting that the sum of the final column is equal to the sum of the last row, we can obtain the following important relation (cf. Tobin [38]):

Table 1 The Summary of the Balance Sheets

Assets \ Sectors	Public	Commercial Banks	Central Bank	Exogenous Supply
Currency	L_c	0	$-M_c$	0
Deposits	L_d	$-M_d$	0	0
Bank Reserves	0	R^b	$-R$	0
Securities (bonds)	S^p	S^b	S^c	D
Equities	pV	0	0	pK
Net Worth	pA	0	0	$D+pK$

(1) $D+pK \equiv pA$,

i. e., the public's nominal net worth pA is equal to the sum of the value of the physical stock of capital, pK, and the total government debt outstanding[3]. We may note that (1) is used in this paper only in the last part of Section III. The assumption that the stock of the accumulated goods is the same in nature as that of current output can be easily relaxed, and is left to the interested reader. Another interesting implication of Table 1, although it will not be used in the present paper, can be obtained by combining the consolidated balance sheet of the commercial banks and the central bank (i. e., $S^b+S^c \equiv M_c+M_d$) to the stock equilibrium conditions $R^b=R$, $S^p+S^b+S^c=D$, and $M_c+M_d=L_c+L_d(\equiv L)$. This yields the relation, $D=L+S^p$, i. e., the public's holding of financial assets $(L+S^p)$ is equal to the national debt.

II. The Money Supply Equation

From the balance sheet of the commercial banks, we obtain

(2) $M_d \equiv \gamma R^b + S^b$, (3) $R^b \equiv \gamma M_d + F$,

where γ and F, respectively, denote the legal reserve requirement ratio and free reserves (excess reserves minus borrowed reserves). We may assume $0<\gamma<1$. Combining (2) and (3), we obtain after some manipulation

3) The discount of holding government bonds by the public and the banks due to the governmental power of taxation will alter this formula, although it will not alter our analysis and results in any essential way as long as such a discount is done at a constant rate.

(4) $M_d = \dfrac{1+\psi}{1+\gamma\psi} R^b$ where $\psi \equiv S^b/F$.

Assume that the public's demand for currency (L_c) is always a fraction of its demand for deposits (L_d), so that $L_c = \alpha L$ and $L_d = (1-\alpha)L$, where $0<\alpha<1$, and $L \equiv L_c + L_d$. The "currency ratio" α is assumed to be exogenous. Then in *equilibrium* in which $L_c = M_c$ and $L_d = M_d$, we have

(5) $M_c = \alpha M$ and $M_d = (1-\alpha)M$, where $M \equiv M_c + M_d$.

From the balance sheet of the central bank, we have

(6) $S^c \equiv M_c + R$.

Hence in equilibrium in which $R^b = R$[4], we obtain

(7) $M = \dfrac{1+\psi}{1+\theta\psi} S^c$ where $\theta \equiv \alpha + (1-\alpha)\gamma$ (so that $0<\gamma<\theta<1$).

The quantity $[M_c + R^b]$ ($\equiv S^c$) is called the *augmented monetary base*[5].

Let i_F and i, respectively, denote the central bank's (nominal) discount rate and the (nominal) rate of interest on securities. We then impose the following behavioral relations on the commercial banks:

(8) $S^b = s^b(i, i_F)(1-\gamma)M_d$ and $F = f(i, i_F)(1-\gamma)M_d$,

so that we obtain $\psi = \psi(i, i_F)$ ($= s^b/f$). We assume

(9) $\partial s^b/\partial i > 0$ and $\partial s^b/\partial i_F < 0$.

Then from (7), we obtain the *Drabicki–Takayama formula*,

(10) $M = \varphi(i, i_F, \gamma; \alpha) S^c$, where $\varphi \equiv (1+\psi)/(1+\theta\psi) > 1$.

It is tempting to call (10) the "supply of money equation." Strictly speaking this is not correct, for in obtaining (10), we used the equilibrium relations (i. e., $L_c = M_c$ and $L_d = M_d$). However, we shall call (10) the *money supply equation*, for it plays such a role in most of the practical applications. Given values of the exogenous variables i_F, γ, α and S^c, (10) describes the relation between M and i, which we call the *money supply curve*.

By using (9), we can obtain the following important relations[6]

(11) $\partial\varphi/\partial i > 0$, $\partial\varphi/\partial i_F < 0$, $\partial\varphi/\partial\gamma < 0$, and $\partial\varphi/\partial\alpha < 0$.

I. e., the money supply curve is upward sloping, and an open market sale, a rise in the discount rate (i_F), or the legal reserve ratio, or a rise in α, *ceteris paribus*, shifts the supply curve to the right, where i is measured on the vertical axis. Also from (10), it is clear that the money supply is unitary elastic with respect to open market operations, i. e., $(\partial M/\partial S^c)(S^c/M) = 1$ (cf. Fand [14]). It is important to realize that S^c in (10) is an exogenous variable, while φ is not. φ may be called the *money multiplier*. It is wrong to assume that φ is constant. Also, being dependent on the rate of interest, φ is not in general an exogenous variable either.

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4) We may assume that R is supplied indefinitely by the central bank with a constant interest rate (i_F). Then $R^b = R$ holds *always*. In some countries, R itself (rather then i_F) is an exogenously controlled policy variable. Japan seems to be an example of such a country. Although we shall not discuss this case, the reader should be able to easily modify the present analysis accordingly.

5) Note that R^b is equal to total bank reserves ($R_T{}^b$) minus borrowed reserves ($R_B{}^b$), so that we have $M_c + R_T{}^b \equiv M_c + R^b + R_B{}^b$. The quantity ($M_c + R_T{}^b$) is called the *monetary base* or the *high powered money*. It is equal to the augumented monetary base only when $R_B{}^b = 0$.

6) We can show that $f + s^b \equiv 1$ by using (2) and (3). See D–T [13].

III. Some Applications

(i) Special Cases:

In the literature, it is often assumed that the commercial banks have no excess reserves and no borrowing from the central bank. In this extreme case, we have

(12) $F=0$.

Recalling $\varphi\equiv(F+S^b)/(F+\theta S^b)$, we have $\varphi\equiv 1/\theta$, so that (10) is simplified to

(13) $M=S^c/\theta$.

Since there are no borrowed reserves by assumption, S^c is equal to the monetary base (cf. fn. 5). Notice that with (13), the money supply curve is independent of the interest rate i (thus vertical), and it is not affected by the central bank's discount policy (i. e., by a change in i_F). If we further assume that there are no central bank notes ($\alpha\equiv 0$), (13) becomes $M=S^c/\gamma$. I. e., the money multiplier is the inverse of the legal reserve ratio.

Suppose $F\neq 0$, but that unemployment is deep and the public's borrowing from the banks is completely interest insensitive. I. e., assume $\partial s^b/\partial i=0$. In this case, we have $\partial f/\partial i=\partial\phi/\partial i=0$ since $f+s^b\equiv 1$, so that

(14) $\partial\varphi/\partial i=0$.

Thus the money supply curve is again a vertical line as in the usual text books[7], and it shifts by the central bank's policy of changing γ, i_F and S^c.

Again suppose $F\neq 0$, but assume that the banks have no borrowing from the central bank. In this case, free reserves are equal to excess reserves, and it is possible and likely that they are completely insensitive to a change of i_F, i. e., $\partial f/\partial i_F=0$. In this case, we can show by using $f+s^b\equiv 1$ that

(15) $\partial\varphi/\partial i_F=0$.

I. e., the money supply is insensitive to the central bank's discount policy.

(ii) Monetary Equilibrium and the Pegging of the Interest Rate:

Write the public's demand function for money L (in the usual manner) as

(16) $L=L(i, pY, pA)$,

where pY denotes the nominal income, and where the function L is homogenous of degree one in pY and pA. The monetary equilibrium can then be described by

(17) $L(i, pY, pA)=\varphi(i, i_F, \gamma; \alpha)S^c$, or (17′) $L(i, Y, A)=\varphi(i, i_F, \gamma; \alpha)S^c/p$ $(=M/p)$,

where we may assume

(18) $\partial L/\partial i<0$, $\partial L/\partial(pY)>0$, $\partial L/\partial(pA)>0$.

Assume that the speed of adjustment in the money market is very fast or infinity. Then the monetary equilibrium is brought about instantaneously before the adjustments in the other markets take place (cf. Brainard and Tobin [3]). I. e., (17) determines the equilibrium interest rate for a given value of pY, pA, i_F, γ, α, and S^c. Suppose that the economy faces inflation and assume that the values pY and pA increase. With the increase in these values, the demand for money increases, and thus the equilibrium money supply (as well as the equilibrium interest rate) increases even in the complete absence of monetary interventions by the central bank, which is illustrated in Figure 1 (by

7) This may justify the Keynesian practice of an exogenous money supply in situations of deep unemployment.

Figure 1 Monetary Equilibrium

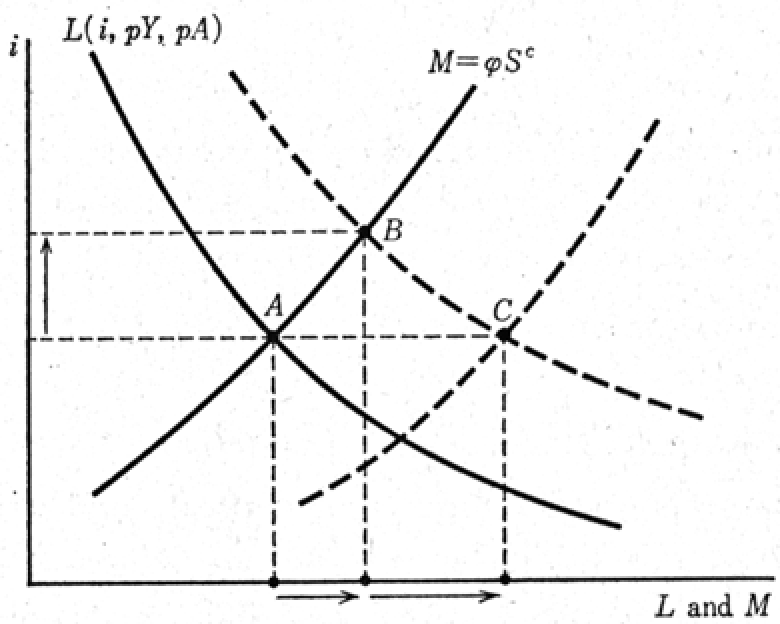
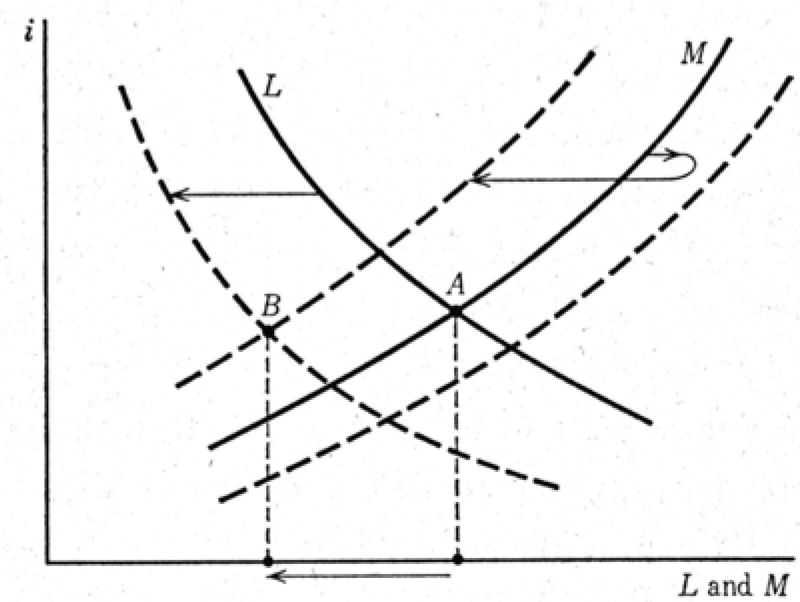


Figure 2 The Great Contraction



the movement from point A to B). If the money supply curve is vertical as in many textbooks, an increase in the money supply cannot occur in this case.

Sometimes, the Treasury wishes to peg the interest rate (or more specifically the price of government bonds), which the central bank agrees to pursue by changing some of the monetary policy parameters (S^c, i_F, and γ). This can happen in the middle of an inflationary period (as was the case in the U. S. in the late 1940's and the late 1960's). In this case, the equilibrium money supply is further increased as indicated by point C in Figure 1. Notice that in the absence of such a policy an increase in the interest rate as indicated by point B would have had a *built–in–flexibility effect* on inflation by discouraging investment and possibly consumption. But with this policy of interest rate pegging, such an effect will be annihilated.

(iii) A Change in α:

The ratio α is predominantly determined by institutional factors, and it would probably be best to treat it as an exogenous variable. It is possible that α sometimes changes drastically, although it may stay constant for a long period of time. A remarkable example is the case of the Great Depression in 1929, in which α increased sharply. For example, in the U. S., the ratio of currency held by the public to bank deposits in July 1960 was at approximately the same level as in July 1932, which in turn was nearly twice as high as in July 1929 (cf. Kaldor [22], p. 13). The great depression contracted the values of pY and pA and thus reduced the demand for money. Under these situations, it is possible that the money supply in the U. S. decreased *despite* expansionary monetary policies[8]. We illustrate this in Figure 2 (assuming away a liquidity trap)[9].

(iv) The Endogenous Supply of Money and the LM-curve:

The LM–curve is defined as the locus of i and Y combinations such that the monetary equilibri-

8) Kaldor writes, "the amount of 'high powered money' in the U. S. increased throughout the Great Contraction of 1929–32……The Great Contraction of the money supply (by one third) occurred *despite* the rise in the monetary base" ([22], p. 13).

9) The L–curve shifts to the left with the decrease in pY and pA. The M–curve shifts to the left drastically due to a drastic rise in α, which more than offsets its rightward shift arising from expansionary monetary policies. The monetary equilibrium point thus moves from A to B, resulting in a decrease in the (equilibrium) money supply.

um relation (17′) holds for given values of p, A, i_F, γ, S^c and α. Assume that α is constant and that monetary policy is undertaken solely by open market operations so that i_F and γ are also constant. Then the total differentiation of (17′) yields

(19) $(\varphi_i S^c/p - L_i)di = L_Y dY + L_A dA + M dp/p^2 + M dS^c/(S^c p)$,

where the partial derivatives are denoted by the subscripts (e. g., $\varphi_i \equiv \partial\varphi/\partial i$). But by the differentiation of (1) (with a constant K), we have $dA = dD/p - D dp/p^2$, which combined with (19) yields

(20) $(\varphi_i S^c/p - L_i)di = L_Y dY + M dS^c/(S^c p) + (M - L_A D)dp/p^2 + L_A dD/p$.

From this it is apparent that the LM–curve is positively sloped (if L_i is finite), and it shifts to the left (right) by an open market sale (purchase). These results correspond to the ones in which the money supply is exogenous[10]. A difference arises for a change in the price level. I. e., an increase in p may not shift the LM–curve to the left. The usual result that the LM–curve shifts to the left (right) by an increase (a decrease) in p (with a finite L_i) occurs if and only if

(21) $L_A < M/D$,

where we may assume $0 < L_A < 1$.

Note that the consolidation of the balance sheets of the commercial banks and the central bank yields $S^b + S^c \equiv M$. Combining this with the stock equilibrium condition for securities $S^p + S^b + S^c = D$, we obtain $M = D - S^p$. Hence if the public is a net borrower ($S^p < 0$), i. e., if its borrowing from commercial banks exceeds its holding of government securities, then we always have $D < M$ and condition (21) is satisfied.

Note that (20) also reveals the effect of a change in D on the *LM*–curve. Starting from the initial balanced budget situation, if government spending is increased by borrowing from the public (bond financing), D increases, which results in a leftward shift of the *LM*–curve, offsetting (at least partially) the expansionary effect of the increased spending.

IV. A Brief Survey of the Endogenous Supply of Money

Once the endogeneity of the money supply is realized, a natural way to incorporate it is to construct a general equilibrium model of financial markets which then simultaneously determine the equilibrium values of the volume, as well as the respective rates of return, of various financial assets (demand deposits, time deposits, Treasury bills, etc.). The equilibrium quantity of money is thus determined by a system of simultaneous equations. This approach, which may be termed as the *Yale approach* (see Brainard and Tobin [3], for example), has a number of obvious advantages. For example, one need not worry about whether or not we should attach a special importance to money compared to other assets; i. e., the assessment of the importance of the "New View" (Gurley–Shaw [17] and others) becomes an empirical question, being incorporated into a more general theoretical framework. Secondly, one does not have to worry greatly about the definition of money, i. e., questions such as whether or not money should include time deposits.

However, this Yale approach, though it will no doubt remain as an important mode of analysis[11],

10) Also, the *LM*–curve becomes flat in the liquidity trap ($L_i = -\infty$), which again corresponds to the case in which the money supply is exogenous. As is clear from (20), the slope of the LM–curve is given by $di/dY = L_Y/(\varphi_i S^c/p - L_i)$.

11) Quite apart from the Yale approach, there is the development of large scale econometric models, in which the financial sector is estimated as a part of such models (cf. de Leeuw [12], Goldfeld [16], Modigliani–Rasche–Cooper [25]. The role of the money supply behavior as an aggregate quantity is left ambiguous in most

has one important weakness. I. e., the quantities such as the demand and supply of money, the rate of interest, etc. are found only by solving a number of these simultaneous equations, and we thus lose a nice simple analytical device of the demand and supply of money (which provides an insight and intuition to the working of the macro system), as is developed and nurtured in the usual litetature and textbooks on macro economics especially those after the "Keynesian revolution." Our effort in section I was thus to find a single equation which summarizes the supply behavior of money, which then avoids the above weakness of the Yale approach.

On the other hand, as is well-known, there are a considerable number of works which attempt to find a simple framework (say a single equation) which summarize the supply behavior of money, many of which explicitly recognize the endogeneity of the money supply. Our task in the this section is therefore to survey briefly but in a unified fashion the diversified works in this non-Yale approach, and to place our work in the previous sections in a proper perspective[12].

For this task, we believe that it is crucially important to have a clear recognition of the balance sheets of the commercial banks and of the central bank. In table 2 we may thus remind the reader of these balance sheets with the notations used earlier. For simplicity, we again assume away time deposits and vault cash, although they are explicity used in some works.

Table 2 The Balance Sheets of the Commercial Banks and the Central Bank

The Commercial Banks (Consolidated)		The Central Bank	
$R^b(\equiv R_T{}^b - R_B{}^b)$	M_d	S^c	M_c
S^b			R

In essence, our approach as summarized in section I utilizes *both* of these balance sheets and incorporates the commercial banks' behavior on free reserves and their demand for earning assets (S^b). Unfortunately, the approaches in the literature (of the non-Yale variety), which we presently classify into four varieties, fall short of this level of generality.

Approach I derives the supply of money equation without utilizing either the balance sheet of the (commercial) banks or of the central bank. In fact, it is based largely on the definitions of money and total reserves. However, this approach explicitly incorporates the importance of the commercial banks' free reserve behavior. This approach is probably best represented by Teigen [32]. Meade's pioneering work [23], as well as Polak-White [29] and Johnson [20] may be classified into this approach, although some of these do not incorporate the banks' free reserve behavior.

Approach II, which is used by Friedman-Schwartz [15] and Cagan [10], again does not utilize either the balance sheets of the banks or of the central bank. Rather, it is solely based on the definitions of money and high powered money. No specification of the banks' behavior is made, or even the banks' free reserve behavior (unlike the previous approach).

Approach III utilizes the balance sheet of the central bank (but not of the commercial banks). This approach, which was used by Teigen in his more recent work [34], is an improvement of approach I. Like approach I, this approach also incorporates the commercial banks' behavior for free reserves, and thus it may be related to the tradition of the "Federal Reserve Conjectures" as publicly discussed

of these studies.

12) We do not wish to attempt a comprehensive survey of all the money supply behavior literature. This would exceed the space limit even if we confine ourselves to the literature written in English. Besides, there are interesting works written in non-English languages. For example, for the ones written in Japanese, we may at once recall the works by Y. Suzuki, K. Suzuki, S. Fujino, S. Royama, K. Hamada, and others. In this sense, our effort here is incomplete.

by Burgess [9], Currie [11], and others in the 1920's and the early 1930's.

Approach IV, which is used by Brunner–Meltzer [7] and Takahashi [31], utilizes the balance sheet of the commercial banks as the basic framework. It also specifies the commercial banks' behavior for earning assets in addition to their free reserve behavior. Our approach in section II follows this approach, except that we explicitly incorporate the central bank's balance sheet in addition to attempting several improvements.

In short, the first two approaches are mainly based on the manipulations of definitions, while the latter two approaches utilize the balance sheet of the central bank or of its commercial banks. Except for approach II, these approaches attempt to incorporate the commercial banks' behavior for free reserves, and in approach IV, also that for earning assets, both of which are at the center of the endogeneity of the money supply. We now summarize these four approaches in turn.

Approach I: This, as represented by Teigen [32], begins with the following definitional relations

$$(22)\quad M \equiv M_c + M_d \equiv \alpha M + M_d,$$

where Teigen explicitly imposes the assumptions of no vault cash and of a constant currency ratio α. Now recalling the definitional relation (3), we have $M_d \equiv (R^b - F)/\gamma$. Substituting this into (22), we obtain Teigen's money supply equation [32]

$$(23)\quad M = \frac{1}{(1-\alpha)\gamma}(R^b - F), \text{ or equivalently}^{13)}, \quad (24)\quad M = \frac{1}{\alpha + (1-\alpha)\gamma}(M_c + R^b - F).$$

The trouble with this approach is that it attempts to explain the behavior of an endogenous variable M in terms of endogenous variables such as R^b and F, where behavioral specifications are not made in a satisfactory manner. We shall discuss this point later in connection with approach III by using his improvement [34] of the money supply equation.

A recent textbook exposition by Johnson [20] can be considered as a special case of Teigen, or the above formula (24), in which both excess and borrowed reserves are assumed to be equal to zero so that free reserves are also equal to zero. Hence substituting F=0 into (24), we obtain Johnson's money supply equation ([20], p. 136)

$$(25)\quad M = \frac{1}{\alpha + (1-\alpha)\gamma}(M_c + R^b) \quad \left(= \frac{1}{(1-\alpha)\gamma} R^b\right).$$

Johnson warns that α may not be constant by recalling the example of the U.S. in the Great Depression.

Actually Johnson's formula is a special case of Meade's formula [23] obtained in 1934, where he also assumes away excess or borrowed reserves. Meade's formula is slighlty more general than Johnson's formula, for he explicitly incorporates vault cash. If we assume away vault cash, then Meade's formula becomes identical to Johnson's fomula[14].

Approach II: This approach has widely been used by Friedman and his associates (the"monetarists"). It is somewhat similar to the previous approach in that it entails nothing but the manipulation

13) Simply add $M_c/(1-\alpha)\gamma$ to the both sides of (23), and recall $M_c = \alpha M$.

14) Let $M_c{}^b$ be the amount of vault cash. Meade [23] assuumes that the ratio $h \equiv R^b/(M_c{}^b + R^b)$ is constant, and obtains his money supply formula, $M = R^b/[(1-\alpha)\gamma h]$, where we assume away his complication due to the existence of gold-reserves. If there is no vault cash, then $h=1$, and Meade's formula reduces to Johnson's formula.

of some definitions, in which the following two definitions are central

(26) $M \equiv M_c + M_d, \ \ H \equiv M_c + R_T^b,$

where H denotes the monetary base or high powered money. From (26), it is apparent that

(27) $$M \equiv \frac{M_c + M_d}{M_c + R_T^b} H.$$

Multiplying the numerator and denominator of (27) by $M_d / R_T^b M_c$ yields

(28) $$M \equiv \frac{(M_d / R_T^b)(1 + M_d / M_c)}{M_d / R_T^b + M_d / M_c} H,$$

which is the Friedman–Schwartz supply of money equation.

From $H \equiv M_c + R_T^b$, we can immediately obtain $M \equiv H / (M_c/M + R_T^b/M)$, which combined with $M \equiv M_c + M_d$ yields Cagan's supply of money equation ([10], p. 12)

(29) $$M \equiv \frac{1}{M_c/M + (R_T^b/M_d)(1 - M_c/M)} H.$$

Both (28) and (29) relate M to the high powered money H by utilizing the two definitions in (26).

Recalling the definition of the "currency ratio" $\alpha \equiv M_c/M$, and defining the "reserve ratio" by

(30) $\gamma^* \equiv R_T^b / M_d,$

we can show that both (28) and (29) are equal to

(31) $$M \equiv \frac{1}{\alpha + (1 - \alpha)\gamma^*} H.$$

I. e., the Friedman–Schwartz and Cagan supply of money equations are identical[15]. We may call (31) the *F–S–C money supply formula*.

As noted earlier, the crux of the above formulation are the two definitions in (26). However, we can carry out an essentially identical analysis by using the augmented monetary base $\bar{H} \equiv M_c + R^b$ instead of the simple monetary base $H \equiv M_c + R_T^b$. I. e., all the equations (27)–(29) hold as they are by replacing R and H by R^b and $\bar{H}$ respectively. Also, defining γ^{**} by

(30′) $\gamma^{**} \equiv R^b / M_d,$

we can rewrite (31) as

(31′) $$M \equiv \frac{1}{\alpha + (1 - \alpha)\gamma^{**}} \bar{H}.$$

Recalling the central bank's balance sheet identity $S_c \equiv M_c + R$ and the equilibrium condition for reserves, $R = R^b$, we have $\bar{H} = S^c$. Then (31′) can be rewritten as

(32) $$M \equiv \frac{1}{\alpha + (1 - \alpha)\gamma^{**}} S^c.$$

Although (32) may readily be obtained from (31), neither Friedman–Schwartz nor Cagan obtained (32) by making these substitutions. Clearly (32) is superior to (31) in the sense that it relates the money supply to the exogenous variable, S^c, rather than to the endogenous variables M_c and R_T^c (which constitute the high-powered money H).

With this modification of the F–S–C formula (31), (32) now looks very similar to our supply equation (10) in the sense that both relate M to S^c. However, there is a crucual distinction between

15) With assumptions of no excess reserves and no borrowed reserves, we have $\gamma = \gamma^*$ as well as $H = M_c + R^b$, so that (31) reduces to Johnson's formual (25). If we interpret Johnson [20] as *not* making these assumptions, then we should call (31), instead of (25), the *Johnson formula* for the money supply equation.

the two. I. e., the formula (32) involves the ambiguity with regard to the question of how the ratio γ^{**} is determined, or more specifically, how the ratio γ^{**} is affected by the commercial banks' interest rate on their loans (i), the central bank's discount ratio i_F, and the legal reserve ratio γ[16]. Our formula (10), on the other hand, systematically shows how these variables affect the money multiplier φ, i. e., the ratio M/S^c, and the relations in (11) completely summarize the relation between φ and the variables such as i, i_F, γ and α.

Approach III: This approach, adopted by Teigen [34], is an improvement of approach I as it explicitly incorporates a "sources and uses" table (of the commercial banks reserves), which in essence is identical to the consolidated balance sheet of the central bank and the Treasury. Assuming a simple Treasury balance sheet so that it can be cancelled out of the sources and uses table, Teigen's new approach amounts to incorporating the central bank's balance sheet identity $S^c \equiv M_c + R$.

Substituting this identity with the equilibrium condition $R = R^b$ into (24), we at once obtain

$$(33) \quad M = \frac{1}{\alpha + (1-\alpha)\gamma}(S^c - F).$$

This formula in essence corresponds to the one obtained by Teigen ([34], p. 95). This is a clear improvement of (23) or (24) in the sense that the endogenous variables such as R^b and M_c are replaced by the exogenous variable S^c.

Teigen [34] then specifies the banks' free reserve behavior by

$$(34) \quad F = F(i, i_F), \text{ with } \partial F/\partial i < 0 \text{ and } \partial F/\partial i_F > 0.$$

Thus (33) may be rewritten as

$$(33') \quad M = \frac{1}{\alpha + (1-\alpha)\gamma}[S^c - F(i, i_F)],$$

so that the money supply M is an increasing function in i, and a decreasing function in i_F, which is consistent with our specification (11). The specification of the banks' behavior as above is an improvement compared to the F–S–C type formula in which no such explicit behaviorial specifications are made.

However, the above Teigen specification might be criticized on two grounds. The first criticism is that Teigen supposes free reserves to be independent both of the legal reserve requirement ratio and of the level of demand deposits M_d. We find this hard to justify. In this connection, we might note an empirical observation that the ratio of free reserves to demand deposits (F/M_d) is a decreasing function of the rate of interest (eg., Polak–White [29] and Meigs [24]). Teigen's specification is not consistent with this observation as it ignores the effect of M_d on the level of F, whereas our specification (8) does.

The second criticism is that the banks' behavior enter the money supply equation solely through their demand for free reserves, and the banks' behavior on earning assets (i. e., the extent of their willingness to make loans) does not affect the money supply. Again, we find this hard to justify.

Next, it might be worthwhile to note that our formula (10) can be obtained from Teigen's formula (33). This can be done by explicitly incorporating the commercial banks' balance sheet identity (2): i. e., with this help it can be shown that (33) can be transformed to (7). Since we observed earlier that

16) In the literature, γ^* (or γ^{**}) is often assumed to be a constant, or at least exogenous. This is rather absurd.

(33) can be obtained from the basic formula (24) of approach I, our formula (10) relates closely to approaches I and III, and it also resembles the F–S–C type formula (32) of approach II.

It might also be worthwhile to note that in a recent textbook exposition of Teigen by Branson ([4], Chapter 13), there seems to be a serious confusion with regard to what constitute the sources and uses table. For this table, Branson offers the identity

$$(35)\quad R_T^b \equiv R_R^b + R_E^b + M_c,$$

where R_R^b and R_E^b, respectively, denote the commercial banks' required reserves and excess reserves. But since we obviously have an identity $R_T^b \equiv R_R^b + R_E^b$, (35) implies $M_c \equiv 0$. Since Branson also assumes $M \equiv \alpha M_c$, $\alpha > 0$, this implies $M \equiv 0$, i. e., money cannot exist in his formulation! Branson, however, is spared from concluding $M \equiv 0$ in his text by failing to take into account the obvious identity $R_T^b \equiv R_R^b + R_E^b$.

Approach IV: This approach, which to our knowledge first appeared in the literature in its crystal form in a difficult article by Brunner–Meltzer [7], probably offers the most important and analytically useful money supply equation. Using (3) and the balance sheet identity (2), we obtain the following equation, which corresponds to B–M's equation (25) ([7], p. 252),

$$(36)\quad M = \frac{1}{\theta + (1-\alpha)g}(M_c + R_T^b - R_B^b)\left[= \frac{1}{\theta + (1-\alpha)g}(M_c + R^b)\right],$$

where $g \equiv F/M_d$ (and $\theta \equiv \alpha + (1-\alpha)\gamma$, as defined earlier). Brunner–Meltzer [7] explicitly specifies the banks' behavior for excess reserves and borrowed reserves as[17)]

$$(37)\quad R_E^b = \mathrm{e}(i, i_F)M_d, \quad R_B^b = b(i, i_F)M_d,$$

with

$$(38)\quad \partial e/\partial i < 0, \quad \partial e/\partial i_F > 0, \quad \partial b/\partial i > 0 \text{ and } \partial b/\partial i_F < 0.$$

Hence, defining the function $g(i, i_F)$ by $g \equiv e - b$, we obtain

$$(39)\quad F = g(i, i_F)M_d, \text{ where } \partial g/\partial i < 0, \ \partial g/\partial i_F > 0,$$

which differs from Teigen's specification (34), but is consistent with the empirical observation that $g \equiv F/M_d$ decreases as i increases. Using (39) and the banks' balance sheet (2), we can obtain

$$(40)\quad S^b = [1 - (\gamma + g)]M_d,$$

so that

$$(41)\quad \psi(\equiv F/S^b) = [1 - (\gamma + g)]/g.$$

Noting (4) and (5) yields

$$(7')\quad M = \frac{1+\psi}{1+\theta\psi}(M_c + R^b),$$

which is essentially a preliminary step used to obtain (7) (cf, D–T [13]). Substituting (41) into this, we at once obtain the Brunner–Meltzer formula (36).

An apparent difficulty in (36) is that all of the components of the augumented monetary base $M_c + R_T^b - R_B^b$ are themselves endogenous variables. However, if we impose the central bank's balance sheet (6) on (36), we obtain

17) Since the commercial banks' reserve behavior (as well as their behavior for making loans) is at the heart of the endogeneity of the money supply, there have been a number of interesting empirical studies recently on the commercial banks' behavior for excess reserves and borrowed reserves (or free reserves). See, for example, Meigs [24], de Leeuw [12], Goldfeld [16], Morrison [26], and Anderson–Burger [1].

$$(42)\quad M=\frac{1}{\theta+(1-\alpha)g}S^c$$

Notice that (42) can also be obtained from the F–S–C type equation (32) by recalling (3)[18]. However (42) is superior to (32) with the explicit functional specification of $g(i, i_F)$ as in (39). Thus, this may be considered as an alternative specification of our formula (10).

An important advantage of our formula (10) compared to the B–M formula (36) is, as mentioned earlier, the use of the central bank's balance sheet, which replaces $(M_c+R_T{}^b-R_B{}^b)$ by S^c. In addition to this, we may list two other advantages.

(a) In Brunner–Meltzer's behaviorial specifications in (37) and (39), the role of the legal reserve requirement ratio is left ambiguous, whereas in our formulation (8) it is made clear since the allocation of S^b and F is interpreted as a portfolio choice of $(1-\gamma)M_d$, the funds available for the commercial banks' discretionary use.

(b) Brunner and Meltzer require the four specifications in (38) with regard to the signs of partial derivatives to obtain their key sign specifications with regard to $\partial g/\partial i$ and $\partial g/\partial i_F$, while we needed only two specification in (9)[19].

Finally, we mention Takahashi [31], which follows B–M [7] closely, especially since he makes a crucial use of the commercial bank's balance sheet, but does not use the central bank's balance sheet. Then he thus obtains (7′) in a more transparent fashion than B–M [7]. Our formulation in section I is no doubt inspired by his method. However, whereas we specify the banks' demand for free reserves and for securities to be functions of i, i_F, γ and M_d as in (8), Takahashi simply assumes

$$(43)\quad S^b=S^b(i) \text{ and } R_E{}^b=R_E{}^b(i), \qquad (44)\quad \partial S^b/\partial i>0 \text{ and } \partial R_E{}^b/\partial i<0.$$

As can easily be seen, (43) is a special case of our (8) and B–M's (37). Furthermore, Takahashi assumes away the banks' borrowed reserves so that $F\equiv R_E{}^b$, which simplifies his derivation of (7′). Notice also that we assumed $\partial s^b/\partial i>0$, *and then* derived $\partial f/\partial i<0$ (which corresponds to Takahashi's $\partial R_E{}^b/\partial i<0$ with $R_E{}^b\equiv F$) by using $f+s^b\equiv 1$, whereas he simply assumes $\partial R_E{}^b/\partial i<0$.

(Akira Takayama: Purdue University • John Z. Drabicki: University of Arizona)

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18) It can also be shown easily that (42) is equivalent to our formula (10), by observing that $g\equiv F/M_d=(1-\gamma)f$ in view of (8). This together with the equivalence of (42) with (10) reveals a close relation between our formulation described in section I and approaches II and IV.

19) Because we can show that $f+s^b\equiv 1$, (9) implies $\partial f/\partial i<0$ and $\partial f/\partial i_F>0$, which is identical to the Brunner–Meltzer specification $\partial g/\partial i<0$ and $\partial g/i_F>0$.

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