Input Choices under Rate of Return Regulation*

Koji Okuguchi

1. *Introduction*

The overview of the discussion given by W. J. Baumol and A. K. Klevorick (1970) on input choices of public utilities under rate of return regulation (i. e. regulatory constraint) did much to enhance our understanding of the essence of the Averch-Johnson proposition first stated in their now classic paper (H. Averch and L. L. Johnson, 1962). Averch and Johnson showed the tendency toward the over-capitalization of monopolies under regulatory constraint.

Using the assumptions of concavity and the complementarity of capital and labor of the revenue function of a regulated firm, A. Takayama (1969) amended the original Averch-Johnson analysis to prove elegantly that the introduction of regulatory constraint would induce the firm to use *both* capital and labor in larger quantities than before regulation. Neither he nor M. El-Hodiri and Takayama (1973), however, were concerned whether the capital-labor ratio under rate of return regulation would be larger than that of no regulation.

Baumol and Klevorick (1970) and F. M. Scherer (1970, Appendix to Chapter 22) recognizing the essential fact that the capital-labor ratio of the firm under regulation is not necessarily larger than that of the firm under no regulation, derived independently a general condition for the larger capital-labor ratio in the case of regulation. In addition to this, Scherer showed that, when the firm has a linear homogeneous Cobb-Douglas production function coupled with a linear market demand function, the tighter the regulation, the more the capital-labor ratio of a regulated firm would diverge from that of an unregulated one and conjectured that the more the production function exhibits increasing returns to scale and the more elastic the demand for the firm, the less the capital-labor ratio of the firm would diverge from that of the unregulated firm Klevorick (1971) also elucidated the role of the degree of returns to scale of a Cobb-Douglas production function in the context of the determination of optimal fair rate of return.

In section 2 of the present paper, we shall reformulate the Baumol-Klevorick-Scherer condition for the emergence of Averch-Johnson phenomena in such a way that the role of degree of returns to scale (i. e. degree of homogeneity) of the neoclassical production function of a regulated firm will become clearer. Our analysis will be greatly facilitated by introducing a new concept "elasticity of marginal revenue with respect to change in supply of output." In section 3, economic interpretations of our condition will be given and the conclusions reached so far by others will be reconsidered in the light of our result.

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2. *The Model and Reformulation of the Baumol-Klevorick-Scherer Condition*

Consider a monopoly firm which is subjected to rate of return regulation. Its identical output Q is a function of two factors, labor L and capital K,

$$Q = F(L, K), \tag{1}$$

which is assumed to satisfy

$$F(L, 0) = F(0, K) = 0,\ F_L > 0,\ F_K > 0 \tag{2}$$

and to be homogeneous of degree m in L and K, where $F_L = \partial F/\partial L$, etc. No assumption is made on the second (cross as well as own) derivatives of F. The price P of and the demand for output of the firm are related by the demand function

$$P = f(Q),\ f' < 0. \tag{3}$$

The prices of labor and capital are denoted by w and r respectively, and $R \equiv PQ$ stands for the revenue function. The firm in question maximizes its profit

$$\pi \equiv R - wL - rK \tag{4}$$

under regulatory constraint

$$R - wL \leq sK \tag{5}$$

where s, which is assumed to be larger than r, is the fair rate of return of the firm permitted by the regulatory agency.

Assuming that the constraint is binding for the problem of rate of return regulation to be essentially relevant to our investigation, we define the Lagrangean expression associated with the above constrained maximization by

$$T \equiv R - wL - rK + \lambda(wL + sK - R),$$

where the Lagrangean multiplier takes a positive value. The first order condition for constrained profit maximization yields

$$(1-\lambda) R_L - (1-\lambda) w = 0 \tag{6}$$

$$(1-\lambda) R_K - r + \lambda s = 0 \tag{7}$$

$$wL + sK - R = 0 \tag{8}$$

and the second order condition

$$\begin{vmatrix} T_{LL} & T_{LK} & T_{L\lambda} \\ T_{KL} & T_{KK} & T_{K\lambda} \\ T_{\lambda L} & T_{\lambda K} & T_{\lambda\lambda} \end{vmatrix} = -(1-\lambda)(s - R_K)^2 R_{LL} > 0, \tag{9}$$

since $\lambda \neq 1$ (this is a consequence of the non-equivalence of unconstrained and constrained profit maximizations) and $R_L = w$ from (6). From (9) it follows that

$$\lambda < 1 \text{ and } R_{LL} < 0,$$

where

$$R_{LL} = R' F_{LL} + R'' F_L^2. \tag{10}$$

The possibility of $\lambda > 1$ must be excluded. For if $\lambda > 1$, the expression for λ obtained from (7) shows $r > s$, which contradicts with our assumption $r < s$. $R_{LL} < 0$ holds true, provided that $F_{LL} < 0$, $R' > 0$, and $R'' < 0$. Sheshinski (1971) introduced an explicit assumption of decreasing marginal revenue, that is, $R'' < 0$.

From the first order condition of constrained maximization (6–8) and the fact that the smaller s is, the tighter the regulation, it follows that $d(K/L)/ds < 0$ must hold for the capital-labor ratio to be an increasing function of tightness of regulation. Baumol and Klevorick (1970, p. 197) and Scherer (1970, p. 554) demonstrated that

$$-KR_{KL}/LR_{LL} < 1 \tag{11}$$

is necessary and sufficient for the validity of $d(K/L)/ds < 0$. This inequality holds certainly in the case where both R_{LL} and R_{KL} are negative. To analyze further the case of $R_{LL} < 0$ and $R_{KL} \geq 0$, we first rewrite (11) as

$$LR_{LL} + KR_{LK} < 0 \tag{12}$$

Taking into account

$$R_{LK} = R' F_{LK} + R'' F_L F_K,$$

(10) and homogeneity of degree m and $m-1$ in L and K of F and F_L, respectively, and applying Euler's theorem on homogeneous function we get;

$$LR_{LL} + KR_{LK} = F_L R' \{m(1-\gamma) - 1\},$$

where by definition

$$\gamma \equiv -Q/R' \cdot dR'/dQ$$

Our reformulation of Baumol-Klevorick-Scherer condition is thus expressed as

$$m(1-\gamma)<1. \tag{13}$$

The inequality shows that the sum of the inverse of the degree of homogeneity of the regulated firm's production function and the elasticity of marginal revenue with respect to change in its supply of output has to be greater than unity.

Before proceeding further, we must analyze R_{LL} in greater detail. To this end, partially differentiate

$$LF_L+KF_K=mF, \tag{14}$$

which is a consequence of Euler's theorem, with respect to L to get

$$F_{LL}=\frac{(m-1)}{L}F_L-\frac{K}{L}F_{KL}. \tag{15}$$

Substituting from (14) and (15) into the definition of the elasticity of factor substitution ρ of the production function

$$\rho\equiv-\frac{F_LF_K(LF_L+KF_K)}{KL(F_{LL}F_K^2-2F_{KL}F_LF_K+F_{KK}F_L^2)}$$
$$=\frac{F_LF_K}{mFF_{LK}-(m-1)F_LF_K},$$

from wihch follows

$$F_{LK}=\frac{\{1+\rho(m-1)\}}{\rho m}\frac{F_LF_K}{F}. \tag{16}$$

Thus from (15) and (16),

$$F_{LL}=\frac{(m-1)F_L}{L}-\frac{KF_LF_K}{LF}\frac{\{1+\rho(m-1)\}}{\rho m}. \tag{17}$$

Rewriting (10) as

$$R_{LL}=R'F_L(F_{LL}/F_L-\gamma F_L/F),$$

and substituting from (17) we see that

$$\operatorname{sgn}R_{LL}=\operatorname{sgn}H(m,\gamma;\theta,\rho), \tag{18}$$

where θ and H are defined by

$$\theta\equiv LF_L/F,$$
$$H\equiv(m-1)-\theta\gamma-(m-\theta)\frac{\{1+\rho(m-1)\}}{\rho m}. \tag{19}$$

We are finally in a position to give economic interpretations of our results.

3. *Interpreting our Results*

In this section we wish to interpret our results (13) and (18), restricting our attention exclusively to the case of $R_{LL}<0$.

Consider first Scherer's case of a Cobb-Douglas production function coupled with a linear market demand function, that is,

$$Q=L^\alpha K^\beta,\qquad P=a-bQ,$$

where α, β, a and b are all positive constants. In this case,

$$\rho=1,\qquad \theta=\alpha$$
$$H=\alpha-1-\alpha\gamma=-\beta-\alpha\gamma<0,$$

implying $R_{LL}<0$. Further as

$$R'=a-2bQ\geq 0\qquad \text{for } Q\leq a/2b,$$
$$R''=-2b,$$

we must have $Q\leq a/4b$ for γ to be less than or equal to one. Assume this to be the case. Then (13) holds certainly if $m\leq 1$. Thus the tighter the regulation, the larger the capital-labor ratio of the firm under regulation provided $m\leq 1$. Moreover, the validity of (13) can not be excluded even in the case of increasing returns to scale, i. e. $m>1$.

As a second illustration we have Takayama's case of a demand function with constant price elasticity combined with a Cobb-Douglas production function,

$$P=AQ^{-\frac{1}{\epsilon}},\ Q=L^\alpha K^\beta,$$

where A, ϵ, α and β are positive constants. In this case, $H<0$ holds as in Scherer's one. Since γ is calculated to equal $1/\epsilon$, (13) becomes

$$m(1-1/\epsilon)<1. \tag{20}$$

Thus other things being equal, an increase in the elasticity of demand has a tendency to violate (13). As was already mentioned in section 1, this tendency was conjectured by Scherer for his special case of Cobb-Douglas and *linear* demand functions. However, our analysis shows that this tendency is observed irrespective of the form of the production function provided that it is neoclassical.

Third, let us consider our results more generally especially in relation to changes in γ and m. Suppose $H<0$, hence $R_{LL}<0$ with (13) holding true. Since $\partial H/\partial\gamma<0$ and (13) remains to hold irrespective of changes in γ, increase in γ does not violate our condition for the emergence of Averch-Johnson phenomena. To see the effects of changes in m, degree of homogeneity, partially differentiate (19) with respect to m to derive

$$\partial H/\partial m = \theta(\rho-1)/\rho m^2,$$

from which

$$\partial H/\partial m \gtreqless 0 \text{ according as } \rho \gtreqless 1.$$

Hence, H is likely to turn out to be positive with increase in m when the elasticity of factor substitution is larger than one. Further, (13) is likely to be violated when m increases. An increase in m is thus shown to have a tendency to violate our reformulated Baumol-Klevorick-Scherer condition independently of the form of the production function so long as it is neoclassical. We may further note that in the case of Cobb-Douglas production function, an increase in m is likely to violate the same condition, which result coincides with Scherer's one for his special case.

(東京都立大学)

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