TECHNICAL PROGRESS AND THE INVESTMENT FUNCTION

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I. *Introduction*

The theory of investment behavior is the field which has been most intensively studied in modern economics, both from the theoretical and empirical points of view, as indicated by, e. g., the vast literature cited by Jorgenson (3). Most of the neoclassical theoreticians have not taken an explicit account of investment behavior in their theoretical framework. They have either postulated that the same economic unit is in charge of both investment and saving decisions, or they have instead supposed that investment opportunities are so abundant that all the savings find their way to be invested regardless of present and future economic conditions.

The neoclassical theory of the firm seems to suffer from a further shortcoming. It is related to the way in which the concept of capital services is employed. In the neoclassical theory, the stock of capital from which the flow of capital services is obtained is regarded as a variable to be freely controlled by the firm. For example, Jorgenson has presented a model in which the desired stock of capital is determined based upon the rate of interest and other market parameters. However, he supposes that the desired level of capital stock may not be instantaneously attained, but instead there exists a time lag in the adjustment process by which the actual capital stock approaches the desired level. Even though some of the empirically found lag patterns have a high degree of statistical fitness, it is rather difficult to find theoretical justification for such a procedure. In fact, if the firm were to have known that it is impossible, for technological or other reasons, to attain the desired level of capital stock, it would certainly take into an explicit account of the lag pattern in determining the desired level of capital stock.

Some authors, such as Lucas (4), Treadway (8), and Uzawa (9) among others, have recently investigated the micro-economic foundation of business investment behavior from a somewhat different point of view. Their approaches have been based upon the concept of adjustment costs along the lines originally introduced by Eisner and Strotz (1). Their basic premises are that the stock of capital is not a variable factor, but a fixed factor of production in the short-run and that the firm incurs costs and requires time in adjusting the level of capital stock. Such assumptions may be regarded as more satisfactory and empirically appealing in the analysis of investment behavior.

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However, their approaches have not taken cognizance of the more crucial role that has been played by technical progress in the dynamic process of the growth of the firm. In the present paper, we should like to extend their analysis in such a way that interactions between technical progress and investment behavior are explicitly examined and some of the more relevant implications upon the pattern of the growth of the firm are derived.

The present study adopts the so-called vintage-capital approach and, in relation to this, we shall choose the level of innovational investment as the unchangeable factor in the short-run, rather than the level of capital stock. The firm then designs an optimal time path of capital stock by adjusting the rate of change in investment. Such an adjustment process is different by one dimension from that of the Lucas-Treadway-Uzawa approach, which makes the level of investment an adjusting variable. Such a framework has been adopted, though without an explicit reference to its theoretical aspects, by a number of empirical studies, such as Jorgenson (3), Griliches and Wallace (2). One of the purposes of the present paper then is to throw some light on the theoretical content of such an empirical relationship in terms of the theory of rational decision of the firm.

In order to clear up this type of investment behavior, we shall introduce an advanced concept with regard to the organizational structure of the firm. As explained in the next section, we shall devide the firm into two organizational sectors; One carries out daily production works, the other is engaged in new innovational investments. It will be argued that the specific character in the latter sector has close relation to the investment behavior above mentioned.

In section II of this paper, we shall construct a framework to analyse the adjustment process of investment under embodied technical progress. In section III, the mathematical analysis of our model will be shown and its economic implications will also be mentioned. Some concluding remarks will be given in the last section.

II. *Adjustment in Investment*

To analyse future plans of firm's investments, we have to specify the internal condition characterizing the physical and organizational structure of the firm and the external condition surrounding the firm.

(i) *Internal conditions*

The restrictive conditions internal for the growth of the firm are closely related with the internal decision processes within the firm. Therefore, it may be desirable, at the outset, to clarify the organizational structure of the firm. Some of the authors referred to above have regarded capital as composed of both physical capacity and managerial ability. Such a view may be interesting and instructive in the sense that it helps us clarify some of conceptual difficulties involved in the process of the growth of the firm. However, in the present paper, we shall adopt a more standard formulation with regards to the production function where capital is interpreted simply as the complex of physical factors of production. This is mainly due to our basic assumptions that technical progress is embodied in physical capital goods. We shall assume that the organizational structure of the firm is conceived of consisting two different sectors. One is the production sector engaged in the management of daily works in production processes, and the other sector is concerned with growth processes of the firm.

The capacity of the first sector may be summarized by the production function the shape of which reflects entrepreneurial or managerial ability endowed in the first sector, as typically assumed in the neoclassical theory of production. Let us consider the representative firm which produces a homogeneous kind of product with productive services, capital services and labor services. We assume that the firm is enjoying technical progress which is flowing exogeneously into the firm. The technical progress is supposed to take a particular type of capital embodiment. Each investment good installed at different times is different in quality. Ruling out the externality among different production processes of different vintages, we can obtain a production function of any investment goods:

$$Y_{t,v}=F(I_v e^{-\mu(t-v)},\ L_{t,v};\ \tau_v). \tag{1}$$

where $Y_{t,v}$: output at time t produced in the production process using the investment goods installed at time $v (t>v)$,

I_v: quantity of investment goods invested at time v,

$L_{t,v}$: labor at time t operating the existing machines of I_v,

τ_v: level of technology of I_v,

μ: constant rate of depreciation.

For simplicity, we assume that the production function is linear homogeneous in capital and labor, and has the "well-behaved" shape. Next, we assume the factor augmenting type of embodied technical progress. As our analysis is not essentially altered by the kind of augmentation, we start by assuming Solow-neutrality of pure capital augmentation in order to make clear-cut following expositions. Thus, equation (1) turns into the form;

$$\begin{aligned} &Y_{t,v}=L_{t,v}f(k_{t,v}), \\ &k_{t,v}=\tau_v I_v e^{-\mu(t-v)}/L_{t,v}, \\ &f'(k)>0, f''(k)<0. \end{aligned} \tag{2}$$

The second sector, referred to as the 'growth sector,' is composed of specific and fixed resources which the firm is unable to change in the short run. The growth processes of the firm are evidently constrained by the quality and quantity of the scarce resources endowed in the growth sector. Such an internal condition, which has been fully described by Penrose (5), is of great, perhaps the greatest, importance for the growth of the firm. For this factor will restrict the dynamic expansion of the firm even in the absence of external constraints and in the presence of the constant return to scale.

We focus our attention upon the following three points which express the essential aspect of the growth sector of a growing firm:

(a) At each point of time, the level of investment has been previously determined. We call this "the fixity of investment."

(b) At each moment, the growth sector of the firm possesses given amounts of resources, and designs and executes an investment plan by means of these resources. We call them "growth resources."

(c) It costs the firm both material and time to increase the magnitude of growth resources.

First, let us examine the fixity of investment. As technical progress is embodied in investment goods, an investment is necessarily endowed with an innovational character, An innovational investment is mostly considered to need fairly difficult arrangements so that the investment can realize its function at full capacity. Before new machines are installed, the firm has to determine the land required for a new factory, has to order new machine and its accessories from a foreign firm, has to master the operating manner of them, and so on. We assume that such innovational investment is fixed in the short run, and is independent of the present rate of interest and other market parameters. Thus, in our model, the initial level of investment is given together with a set of past machines which has already worked in production processes.

$$I_0 = \text{given} \tag{3}$$

Second, let us specify the relation between investment and growth resources. For the sake of simplicity, we assume that:

(a) The required stock of growth resources is proportional to the level of actual investment.

(b) At the initial point of time, the growth sector has held the stock of growth resources which is required for given investment I_0.

(c) The growth sector possesses the required stock at all times and does not reserve a slack of growth resources.

In fact, the planning and practice of investment might perhaps require growth resources in a non-linear form and need other inputs such as so-called forgone outputs. The growth sector will usually own some slack of growth resources in order to respond quickly to the changes in investment opportunities. But, we assume the relation shown below, for it simplify our analysis without losing sight of the essence of the growth mechanism of the firm.

$$\alpha I_t = M_t, \text{ for all } t \geqslant 0. \tag{4}$$

where M_t is the stock of growth resources and α is a positive constant.

Third, we introduce costs of adjustment in the stock of growth resources. Since growth resources are considered specific for the individual firm, the adjustment costs of them cannot be defined merely in technological terms such as a gestation period, a delivery period, etc. They must include organizational costs which transforms a common input into an increase in specific resources. We think that these costs of specific character is the most fundamental to the growth of the firm as an organic system containing the managerial or entrepreneurial abilities. In this respect, Uzawa (9) has formulated, in a somewhat different context from this paper, a cost function relating these abilities. He called it "the Penrose Function." We also adopt the function in order to express the adjustment costs in growth resources.

$$C_t = M_t \varphi(\dot{M}_t / M_t),\ \varphi'(\cdot) > 0,\ \varphi''(\cdot) > 0. \tag{5}$$

where C : adjustment costs in terms of output involved in the changes in growth resources

M : stock of growth resources.

Combining equation (4) with equation (5), we obtain

$$C = \alpha I_t \varphi(\dot{I}_t / I_t). \tag{6}$$

(ii) ***External Conditions***

We shall treat only a situation of perfect competition, where the firm must act as a pure price-taker in all markets of product, investment goods, labor and money funds. At the initial time point 0 when the planning at issue is going to be decided, these competitive markets announce to the firm various prices such as a product price, price of investment good, wage rate, and interest rate. The firm has to form expectations about future prices in order to calculate the future returns and the future costs of any investment plan. We assume a static expectation, except about the price of investment goods. If the economy is not in a stationary state, it will be appropriate to suppose that the firm expects future prices to change in various ways, the directions of which are indicated by past informations. But, in this paper, we sacrifice the realism about expectations so that we may explicitly analyse more essential aspects of investment behavior. This implies that the firm expects the present prices of output, labor, and money fund to continue in the future at the same level. As for the expectations concerning prices of investment goods, a dynamic consideration will be required. As we shall suppose later the firm to expect the quality of investment goods to progress, it will be natural that the firm should expect prices of investment goods to rise along with the improvements in its quality. Moreover, being given the expectations about future technology, expected prices of investment goods are naturally supposed to be higher if the present prices are higher. Thus, we assume following conditions:

$$\left.\begin{array}{l} \omega_t^e = \omega_0,\ \delta_t^e = \delta_0 \\ q_t^e = q(q_0, \tau_t^e) \end{array}\right\} \tag{7}$$

where ω : rate of wage in terms of the price of product,

δ : rate of interest,

q : price of the investment goods in terms of the price of product,

x_t^e : expected value at the present time about x at time t.

and we assume following conditions:

$$\left.\begin{array}{l} \dfrac{\partial q}{\partial \tau_t^e} > 0 \quad \dfrac{\partial q}{\partial q_0} > 0 \quad q(q_0, \tau_t^e) = q_0 \\ q < \hat{q},\ \ \hat{q} = \lim\limits_{t \to \infty} q(q_0, \tau_t^e) \end{array}\right\} \tag{8}$$

The firm has also to form the expectations concerning future levels of technology, $\tau_t^e = \tau(t)$, which will be embodied in newly produced machines. In reality, there exist some technical progresses generated inside the firm. They are produced by employing various resources in the R&D sector of the firm. But, in this paper, we assume that there are no R&D activities.

Alternatively, we may assume that if there are R&D activities inside the firm, the costs are proportional to the level of technology and can be included in the prices of investment goods. At any rate, the expectations about exogeneous technical progress will play an important role in our model. Next, we impose two qualifications concerning the expectations on future technology:

(a) The future level of technology is anticipated to have some upper limit.

(b) The time path of expectation may take various shapes at comparatively early stages, but ultimately, approaches asymptotically the upper limit.

A few cases are illustrated in Fig. 1.

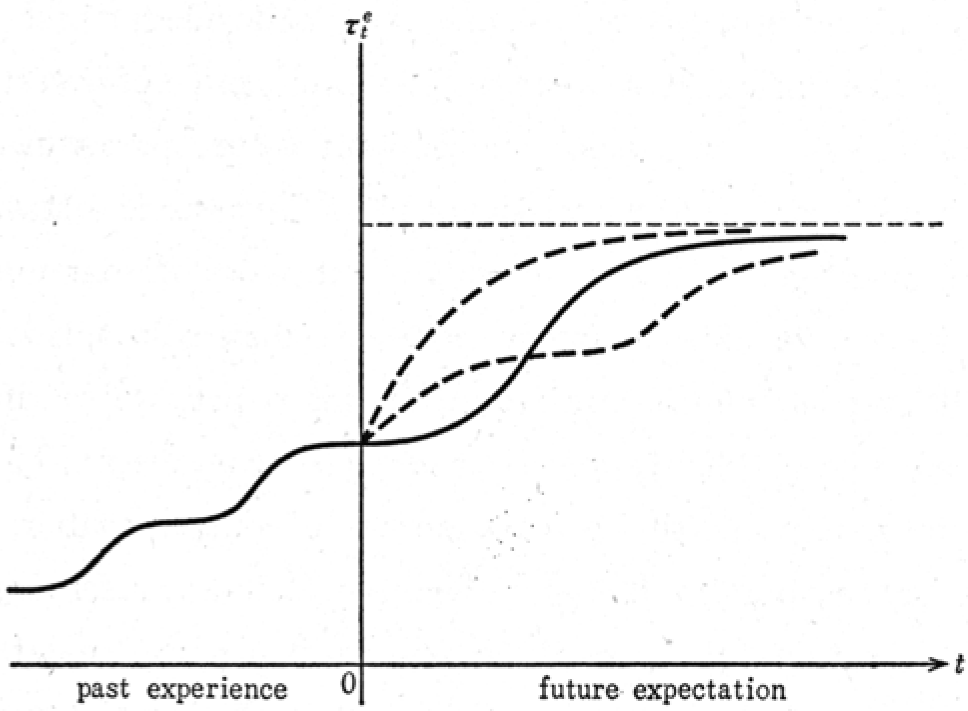

Figure 1

We believe that the existence of some upper limit to the technological expectation is reasonable from both theoretical and empirical points of view. The concept of technology should be defined as having a concrete substance, at least in the case of economic production. Especially when technology is embodied in investment goods, technical progress should be anticipated in the concrete form, for example, a hand-operated lathe, an electric lathe, or an atomic lathe. So long as the firm has now only a limited amount of technological information, it will be unrealistic for the firm to expect an infinite level of technology worthy of economic considerations. In addition to this respect, if aspects of uncertainties are taken into account in determining expectations, we shall have a definite reason to set some upper limit to it. Thus, we assume

$$\left.\begin{aligned} &\tau_t{}^e = \tau(t), \\ &\tau(t) < \hat{\tau}, \quad \lim_{t\to\infty} \tau(t) = \hat{\tau}, \\ &\tau'(t) > 0, \\ &\tau''(t) < 0 \quad \text{for } t > \text{some large } t \end{aligned}\right\} \tag{9}$$

Finally, we assume that the end of the firm is to maximize the present value of its own. Under our assumption of no externality among factories or machines of different vintages, the firm has to maximize the present value of new factories or machines built from now on. The objective function to be maximized takes the form:

$$\int_0^\infty \left[\int_v^s Q_{t,v} e^{-\delta_0 t} dt - q_v{}^e I_v e^{-\delta_0 v} - C_v e^{-\delta_0 v} \right] dv. \tag{10}$$

where all variables are expected at time 0, and

$Q_{t,v}$: quasi-rent generated at time t from the production process using machines with vintage v

s : time point at which machines with vintage v is expected to be obsolete.

Restrictive conditions consist of all equations from (1) to (9).

III. *The Optimal Investment Decision Under the Embodied Technical Progress*

Let us first consider the quasi-rent. From relations (1), (2), (3) and (4),

$$Q_{t,v} = \{ L_{t,v} f(k_{t,v}) - \omega_0 L_{t,v} \} \tag{11}$$

It is clear that the firm designs to hold constant the capital intensity measured in efficiency units, $k_{t,v} = \tau_v I_v e^{-\mu(t-v)} / L_{t,v}$ for the real rate of wage is expected to continue at the present level. Namely, the next relation, which means the equality between the marginal product of labor and the real wage, should be satisfied to maximize the quasi-rent at each time $t > v$.

$$f(k_{t,v}) - k_{t,v} f'(k_{t,v}) = \omega_0. \tag{12}$$

Rewriting this, we can obtain

$$k_{t,v} = k(\omega_0), \quad k'(\omega_0) > 0. \tag{13}$$

As the production process of vintage v is expected at time 0 to produce a positive quasi-rent at all future dates, the investment goods of vintage v is expected not being obsolete forever. Thus, from (7), (6) and (11), our objective function (10) can be expressed as follows;

$$\int_0^\infty [A(\omega_0, \delta_0) \tau(v) I_v - q(q_0, \tau(v)) \mathrm{I}_v - \alpha I_v \varphi(z_v)] e^{-\delta_0 v} dv. \tag{14}$$

where

$$\left. \begin{aligned} A(\omega_0, \delta_0) &= \frac{f(k(\omega_0)) - \omega_0}{(\delta_0 + \mu) k(\omega_0)}, \\ z_v &= \dot{I}_v / I_v. \end{aligned} \right\} \tag{15}$$

and a dot denotes the derivative with respect to v. Our constraints are summarized as follows,

$$\dot{I}_0 = \text{given}, \quad \dot{I}_v = z_v I_v. \tag{16}$$

To find the optimal control $z_v{}^*$ and the optimal plan of future investment $I_v{}^*$, we have to examine next necessary conditions derived from the mathematical technique of the Maximum Principle (6).

$$\left. \begin{aligned} &\alpha \varphi'(z) = \lambda, \\ &\dot{\lambda} = (\delta_0 - z) \lambda + \alpha \varphi(z) - \{ A(\omega_0, \delta_0) \tau(v) - q(q_0, \tau(v)) \} \end{aligned} \right\} \tag{17}$$

In these relations, λ can be interpreted as the shadow or demand price of investment goods, which is explicitly shown below. On the other hand, $\alpha \varphi'(z)$ is equal to $\partial C / \partial \dot{I}$, namely the marginal cost of investment. Accordingly, the first equation in (17) implies the subjective equilibrium between the cost and the shadow price of investment. From this equation and equation (5), we can write

$$z = z\left(\frac{\lambda}{\alpha}\right), \quad z'\left(\frac{\lambda}{\alpha}\right) > 0 \tag{18}$$

The second equation of (17) can be solved into the following

$$\lambda_v = \int_v^{\infty} [\{Q_{v,v} - q_v I_v - C_v\}/I_v] e^{-\int_v^z (\delta_0 - z_x) dx} dz \tag{19}$$

This expresses distinctly the meaning of λ as the shadow price of investment. Alternatively, we can rewrite this into

$$\frac{A(\omega_0, \delta_0)\tau(v) - q(q_0, \tau(v)) - \alpha\varphi(z)}{\lambda} + \frac{\dot{\lambda}}{\lambda} = \delta - z.$$

The left-hand side is regarded, by the analogy of the competitive capital market, as the sum of the net cash flow per unit of investment and capital gain. This value must be equal to the rate of interest adjusted by the growth rate of the firm which is shown by the right-hand side. Combining the second equation of (17) and (18), we obtain the next non-autonomous differential equation:

$$\dot{\lambda} = \left\{\delta_0 - z\left(\frac{\lambda}{\alpha}\right)\right\}\lambda + \alpha\varphi\left(z\left(\frac{\lambda}{\alpha}\right)\right) - [A(\omega_0, \delta_0)\tau(v) - q(q_0, \tau(v))]. \tag{20}$$

In analysing the nature of paths of equation (20), (λ, v), we suppose the following situation which seems worthy of explicit examination. Let us consider the last term of (20).

$$T(v{:}\alpha, \omega_0, \delta_0, q_0) = A(\omega_0, \delta_0)\tau(v) - q(q_0, \delta(v)). \tag{21}$$

We assume that the time shape of this expression $T(v)$ is similar to the expectation about future technology in next two points:

(a) $T(v)$ has an upper limit, (b) $T(v)$ asymptotically approaches the upper limit, at least, after the passage of a sufficiently long time period. Such a situation is most likely to happen under our assumption about the expectation with regards to the prices of investment goods and the future levels of technology, which are shown in (8) and (9), respectively. A few typical shapes of $T(v)$ are illustrated in Fig. 2. Next, we check the character the function made up of first two terms of equation (20).

$$\psi(\lambda) = \left[\delta_0 - z\left(\frac{\lambda}{\alpha}\right)\right]\lambda + \alpha\varphi\left(z\left(\frac{\lambda}{\alpha}\right)\right). \tag{22}$$

Differentiating this, and from (17), we obtain

$$\psi'(\lambda) = \delta_0 - z\left(\frac{\lambda}{\alpha}\right) > 0. \tag{23}$$

In the following analysis, we treat the most typical case in which the rate of interest is high enough to allow us to take for granted a positive sign of $\delta_0 - z\left(\frac{\lambda}{\alpha}\right)$. (If the matters are to the contrary, we can guess intuitively that all investments are postponed until the infinite future).

From (20), (21) and (23), we can find,

$$\left.\frac{d\lambda}{dv}\right|_{\dot{\lambda}=0}=\frac{T'(v)}{\delta_0-z\left(\dfrac{\lambda}{\alpha}\right)}. \tag{24}$$

Thus, the curvature of $\dot{\lambda}=0$ curve is governed only by the sign of $T'(v)$, as shown in the following figure.

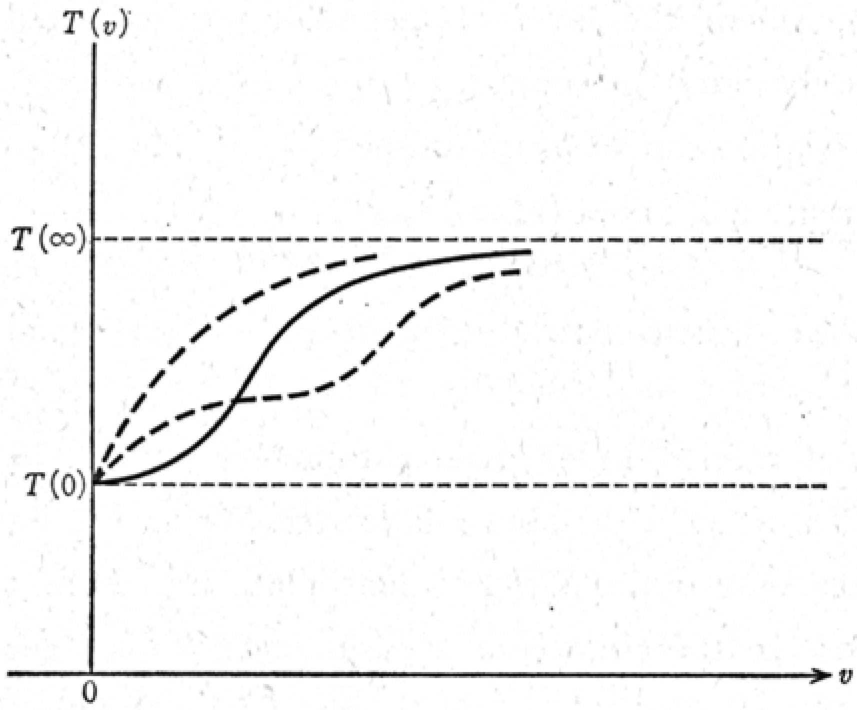

Figure 2

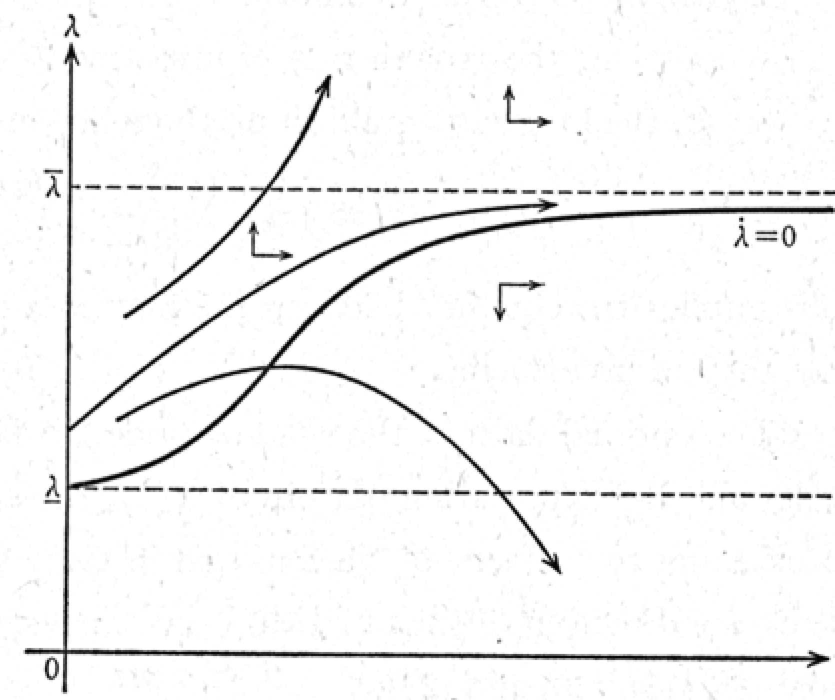

Figure 3

where

$$T(\infty)=\phi(\bar{\lambda}),\ \text{ and } T(0)=\phi(\underline{\lambda}).$$

We can easily show that there exists a unique path (λ^*, v) which satisfies the condition $\lim_{v\to\infty}\lambda_v=\bar{\lambda}$, and all other paths depart away from this path as indicated in the above figure. The reasoning is as follows;

(a) It is self-evident that there are at least one path going into the region between $\bar{\lambda}$ line and $\dot{\lambda}=0$ curve. (b) The distance between those two curves becomes narrow by degrees. (c) Accordingly, if there exist multiple paths approaching $\bar{\lambda}$ line, the upper one must have a flatter slope, at least after the passage of a sufficiently long period. (d) But, this contradicts the condition

$$\partial\dot{\lambda}/\partial\lambda=\phi'(\lambda)=\delta-z>0. \tag{25}$$

Thus, we can find that the differential equation of the shadow price (20) has a unique stable branch approaching $\bar{\lambda}$. But, the so-called transversality condition $\lim_{v\to\infty}\lambda e^{-\delta_0 v}=0$ is not a necessary conditon for optimality in an infinite-time programming. It relates to a sufficient condition shown below

$$\lim_{v\to\infty}\lambda_v e^{-\delta_0 v}I_v=0. \tag{26}$$

As we are now supposing the situation $\delta_0>z=\dot{I}_v/I_v$, the stable branch of the shadow price surely meets this sufficient condition for optimality. We can call the stable branch "the most reliable optimal path of shadow price," or we may be courageous enough to call it "the optimal path," in analogies to the problem of finite-time horizons.

From the foregoing analysis we have obtained the optimal plan of investment,

$$\left(\frac{\dot{I}_v}{I_v}\right)^* = z_v{}^* = z^*\left(\frac{\lambda_v{}^*}{\alpha}\right). \tag{27}$$

We should note some characters of the optimal plan:

(a) The growth rate of investment $z_v{}^*$ increases gradually, and approaches an upper limit $z^*(\bar{\lambda})$, under the embodied technical progress.

(b) In the long run equilibrium of the firm, where the level of technology is expected constant, the growth rate of investment also becomes constant.

(c) In the long run equilibrium, the following relation should be established

$$\alpha\varphi'(z(\infty)) = \frac{T(\infty) - \alpha\varphi(z(\infty))}{\delta_0 - v(\infty)} = \frac{Q_{\infty,\infty} - \alpha\varphi(z(\infty))I_\infty}{[\delta_0 - z(\infty)]I_\infty}. \tag{28}$$

This implies the equality between the marginal cost of investment adjustment and the marginal efficiency of investment.

The optimal path of the shadow price $\lambda_v{}^*$ is determined under given parameters such as ω_0 δ_0, q_0, and the expectation pattern of $\tau(v)$, the last of which is denoted τ hereafter. We set about to examine the effects of various parameters upon the optimal investment plan. Let us first check the direction of shift of $\dot{\lambda}=0$ curve in response to the change in a parameter. We can see from (8), (15), (20) and (23),

$$\left.\begin{aligned}
\left.\frac{\partial\lambda}{\partial\delta_0}\right|_{\substack{\dot{\lambda}=0\\ v=\text{const.}}} &= \frac{\tau(v)\cdot\partial A(\omega_0,\delta_0)/\partial\delta_0 - \lambda}{\delta_0 - z} < 0\\
\left.\frac{\partial\lambda}{\partial\omega_0}\right|_{\substack{\dot{\lambda}=0\\ v=\text{const.}}} &= \frac{\tau(v)^{\partial A(\omega_0,\delta_0)}/\partial\omega_0}{z - {}^0\varrho} < 0\\
\left.\frac{\partial\lambda}{\partial q_0}\right|_{\substack{\dot{\lambda}=0\\ v=\text{const.}}} &= \frac{-\partial q(q_0,\tau(v))/\partial q_0}{\delta_0 - z} < 0
\end{aligned}\right\} \tag{29}$$

From above, we can find that the phase diagram in Figure 3 goes down together with the increase in δ_0, ω_0 or q_0. Figure 4 illustrates two different situation with regard to these parameters. It is easily shown that there never occurs such an intersection of two stable branches as shown in

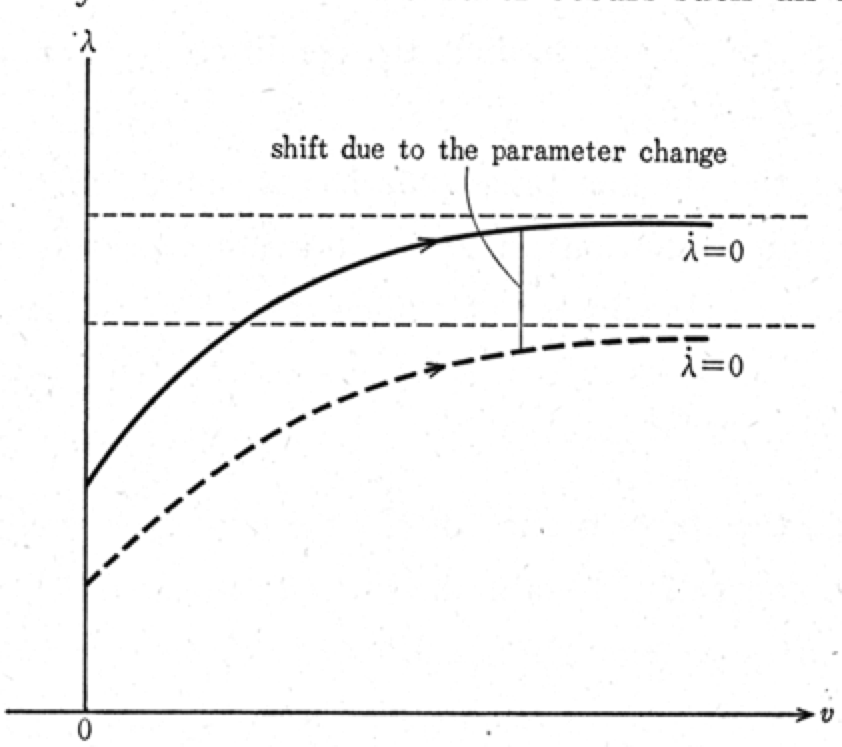

Figure 4

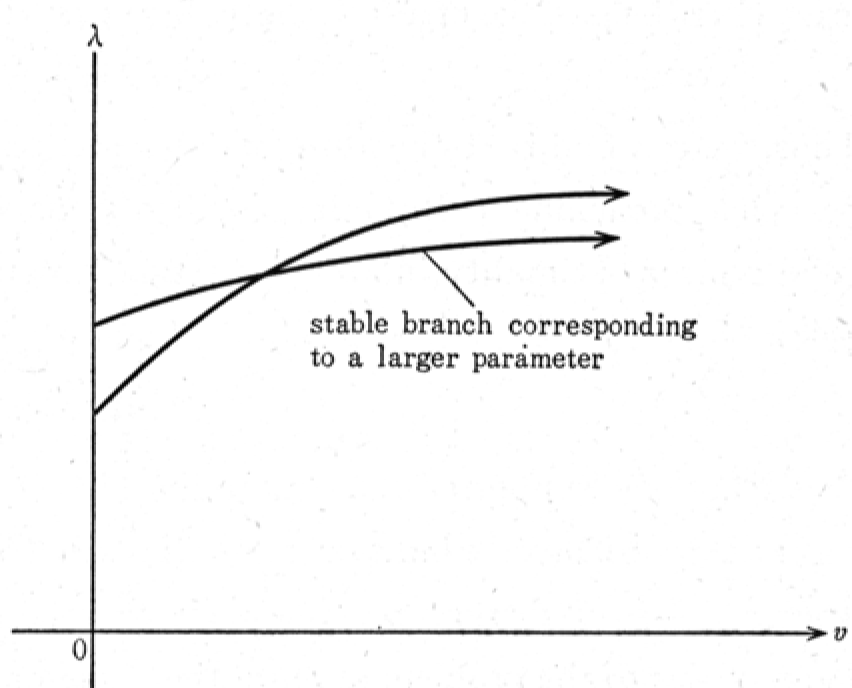

Figure 5

Figure 5. In order for such an intersection to occur, $d\lambda/dv$ has to be smaller at least at the point of intersection in the case corresponding to a larger parameter than in the case corresponding to a smaller parameter. But, we can find:

$$\left.\begin{aligned} \left.\frac{\partial\dot{\lambda}}{\partial\delta_0}\right|_{\substack{\lambda=\text{const.}\\ v=\text{const.}}} &= \lambda - \frac{\partial A}{\partial\delta_0} > 0, \\ \left.\frac{\partial\dot{\lambda}}{\partial\omega_0}\right|_{\substack{\lambda=\text{const.}\\ v=\text{const.}}} &= -\frac{\partial A}{\partial\omega_0} > 0, \\ \left.\frac{\partial\dot{\lambda}}{\partial q_0}\right|_{\substack{\lambda=\text{const.}\\ v=\text{const.}}} &= \frac{\partial q(q_0, \tau(v))}{\partial q_0} > 0. \end{aligned}\right\} \tag{30}$$

Hence, it is impossible for two optimal paths of λ to intersect with each other.

Finally, we have to refer to the effect of the change in technological expectation τ. We have already noted that the shape of $\dot{\lambda}$ curve is governed by that of τ. By the same reasoning as above discussions, it can be concluded that a uniform upward shift in the shape of technological expectation accelerates the rate of change in investment. In the case of ununiform shifts, complicated changes in investment plan will occur as shown in Figure 6.

Thus, we have obtained the rational plan of adjustment in innovational investment under the embodied technical progress. Since all parameters are announced to the firm in markets at the time point of planning, it is enough for us to pay attention to the investment decision at the planning time t. We can write in general,

$$\dot{I}_t/I_t = z(\delta_t, \omega_t, q_t, \tau), \tag{31}$$

where

$$\left.\begin{aligned} \partial z/\partial\delta<0, \quad \partial z/\partial\omega<0, \\ \partial z/\partial q<0, \quad \partial z/\partial\tau>0. \end{aligned}\right\} \tag{32}$$

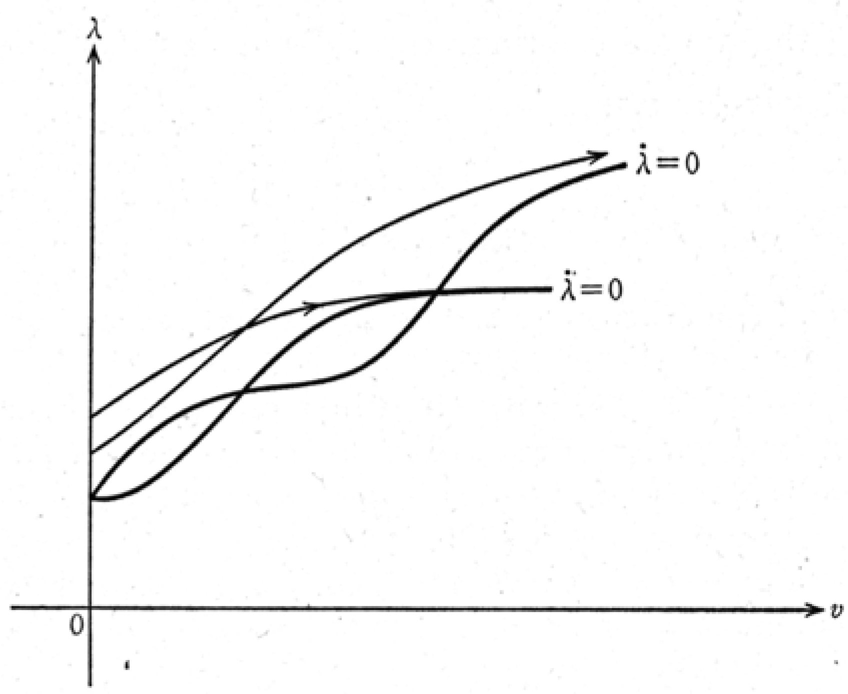

Figure 6

IV. *Concluding Remarks*

This paper mainly concerned with following factors affecting the dynamic planning of the firm: (a) the fixity of innovational investment, (b) the existence of growth resources inside the firm, (c) the embodiment of technical progress in investment goods. We have derived the behavioristic function which explains the change rate of investment by the real rate of interest, the real wage rate, the relative price of investment goods, and the present level and future anticipated course of technology. Three price variables are, naturally, expected to pull down the change rate of investment. The rate of technical progrss are, on the contrary, expected to draw up it.

Many empirical researchers has used the investment functions which contains the lagged level of investment as an important explanatory variable. But they have little inquired into the behavioristic logic of those functions. And further, it has been widely aware that the rate of investment has been accelerated by the rapid rate of technical progress. But, few have shown theoretical and empirical investigations with regard to its mutual relation.

It is sure that we cannot find without difficulty adequate indices of the level of technology and the rate of technological progress. However, as for their first approximations, we may be permitted to use the total productivity and the residual of the productivity change not explained by the change of factor inputs. Our analysis may present a theoretical base for such an empirical consideration into the relation between investments and innovations.

If we regard the dynamic decision about investment and innovation as one of the most important function of the firm, the organizational structure of the firm should not be expressed as a mere agency of prodcution. Our analysis gives an attempt which allows for the real state of the dynamic management of the firm. The magnitude of growth resources is also difficult to be measured, so that we have avoided an explicit representation of growth resources by assuming its proportional relation to the level of investment. But, if managerial data of the firm are to be accumulated, our adjustment cost function will be more rigorously specified. If we succeed this, our model sould be next extended to some directions, such as relaxing the assumption of perfect competition, investigating the dynamic formulation of expectation, and including the R&D or learning process in the inventive activity.

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