THE JAPANESE GROWTH PATH: EQUILIBRIUM OR DISEQUILIBRIUM?

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Introduction

It would be interesting to know whether the Japanese-development success story has involved in the past, or involves in the present, an equilibrium or a disequilibrium expansion path. An equilibrium path, particularly a stable one, could be interpreted as a feather in the cap of a basically market-oriented mechanism. A disequilibrium path, particularly if imposed to counteract the instability of the equilibrium one, would suggest the necessity of continued substantial intervention. Intervention might well be of the historical Japanese type, although other types are equally conceivable.

Most statistical and theoretical models of Japanese economic growth assume at least implicitly a stable equilibrium path, whose detailed parameters varied somewhat over time in response to autonomous shocks, either favorable (technological progress) or unfavorable (wartime destruction). On the other hand, a previous effort of my own ascribed much of Japan's glamorous statistical record to "disequilibrating" intervention for the purpose of "private pyramid-building," i. e., the simultaneous creation of purchasing power and excess capacity, or rather, the creation of excess capacity for the sake of increased purchasing power. The Japanese reaction has been generally unfavorable;[1] I should like to pursue the issue further here, along lines of improved exposition rather than substantive innovation.

Harrod-Domar Models

For the present exposition, as in the earlier one, a Harrod-Domar growth model will suffice, although such simple models have been superseded by neo-classical models, multi-sector models, turnpike theorems, etc., without closing the embarrassing gap between formal growth theories

1) M. Bronfenbrenner, "Economic Miracles and Japan's Income-Doubling Plan," in W. W. Lockwood (ed.), *The State and Economic Enterprise in Japan* (Princeton: Princeton University Press, 1965), ch. xi. (When an abbreviated Japanese-language edition of this volume was prepared, this chapter was among those omitted from translation.) An earlier "disequilibrium" interpretation is contained, however, in Miyohei Shinohara, *Growth and Cycles in the Japanese Economy* (Tokyo: Kinokuniya, 1962), p. 108. Shinohara refers to Japanese experience (with credit to an unpublished study by G. C. Allen) as a "high rate of growth by means of an unstable oscillation of investment and prices, and not as smooth-going a process as is formulated in modern economic growth theory."

and the historical experience of developing countries. I shall begin by reviewing first the Harrod and then the Domar form of the basic model.

The Harrod criterion for equilibrium growth[2] is that *ex ante* (desired) saving and investment should be equal at all points in time. Let *ex ante* saving (S) be a positive fraction (α) of the national income or product (Y). Let *ex ante* investment (I) be proportional to the *growth* of income (dY/dt). The factor of proportionality (β), called an *accelerator*, is also positive but may exceed unity. The condition for equilibrium or equality between *ex ante* saving and investment is:

$$\alpha Y - \frac{\beta dY}{dt} = 0 \text{ or } \frac{dY}{Y} - \frac{\alpha}{\beta} dt = 0$$

The solution of this differential equation is the equilibrium growth rate of income through time:

$$Y = Y_0 e^{\left(\frac{\alpha}{\beta} t\right)} \tag{1}$$

Multiplication by the saving ratio (α) yields, as long as $S=I$ *ex ante:*

$$I = I_0 e^{\left(\frac{\alpha}{\beta} t\right)} \tag{2}$$

The Domar model arrives at a similar result from a different criterion of equilibrium growth, namely, that the income-increasing (multiplier) effect of any increase in investment should balance its output-increasing effects. The multiplier effect, using our previous notation, is (dI/α).[3] The output-increasing effect is ($I\sigma dt$), where (σ) is the marginal efficiency of the new investment, as augmented or diminished by complementarity between capital and labor in the production process.[4]

The fundamental differential equation of the Domar model[5] is:

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2) R. F. Harrod, "An Essay in Dynamic Theory," *E. J.* (March, 1939). This criterion or model should not be confused with the Harrod growth *identity*, which holds in equilibrium and disequilibrium situations alike. If $S=I$ *ex post* (statistically) as it does, if $S=sY$, if $I=dK$ (where (K) is the capital stock), if (G) signifies the income growth rate (dY/Y), and if the capital coefficient (dK/dY) is denoted by (C), we have:

$$\frac{dY}{Y} = \frac{dY}{I}\frac{S}{Y} = s\frac{dY}{dK}$$

whence $G=(s/C)$ or $GC=s$.

3) If we write: $Y=C(Y)+I$, (C) being consumption and functionally related to (Y):

$$Y\left(1-\frac{C(Y)}{Y}\right) = I \text{ and } Y = \frac{I}{1-\frac{C(Y)}{Y}} = \frac{I}{\alpha}$$

It follows that $\frac{dY}{dI} = \frac{I}{\alpha}$ and $dY = \frac{dI}{\alpha}$

4) If we define (N) as the volume of employment, and postulate a production function $Y=F(K, N)$ for a given technology or "state of the arts:"

$$\frac{dY}{dK} = \frac{\partial F}{\partial K} + \frac{\partial F}{\partial N}\frac{dN}{dK} = \sigma$$

The first term on the right, ($\partial F/\partial K$), is the direct marginal efficiency of investment. The other terms embody the indirect complementary effects of that investment.

5) E. D. Domar, "Expansion and Employment," *A. E. R.* (March, 1947), reprinted in Domar, *Essays in the Theory of Economic Growth* (New York: Oxford, 1947), ch. iv.

$$\frac{dI}{\alpha} - I\sigma dt = 0$$

Its solution is:

$$I = I_0 e^{\alpha\sigma t} \tag{3}$$

If $I = S = \alpha Y$—the Harrod equilibrium condition— (3) becomes:

$$Y = Y_0 e^{\alpha\sigma t} \tag{4}$$

Equations (3–4), like equations (1–2), are equilibrium gowth paths. Comparing (1) with (4), or (2) with (3), shows the two paths to be essentially the same. The only difference is that the accelerator (β) of the Harrod model is, in equilibrium, the reciprocal of the capital productivity (σ) in the Domar model. These similarities have inspired the short-hand term "Harrod-Domar" model.

Instability

The Harrod-Domar model is unstable. This means that departure from any of the equilibrium growth paths (1–4) is not self-correcting but self-augmenting. It is recognized that this instability may under certain circumstances disappear, if a variable capital-labor ratio is introduced into the model.[6)]

A simple demonstration of the instability of the Harrod-Domar models is obtained by rewriting any of equations (1–4) as an inequality (involving departure from the equilibrium path), and then reversing the derivation of the equation concerned. For example, in the case of the Domar model, equation (3), and an upward deviation;

$$I > I_0 e^{\alpha\sigma t} \text{ leads to } \frac{dI}{\alpha} > I\sigma \text{ dt}$$

This means that when investment *already* exceeds its equilibrium value, the multiplier effect

6) Compare particularly R. M. Solow, "Contribution to the Theory of Economic Growth, *Q. J. E.* (February, 1956). The development requires that the production function $Y = F(K, N)$ of Note 4, above, be linear homogenous. By the linear-homogeneity property:

$$\frac{Y}{N} = F\left(\frac{K}{N}, 1\right)$$

If we write (y) for (Y/N) and (x) for (K, N), we obtain

$$y = f(x)$$

which is not linear homogenous.

The condition for an equilibrium value (x^e) of the capital-labor ratio (x) is that (dx/dt) vanish:

$$\frac{dx}{dt} = \frac{d}{dt}\left(\frac{K}{N}\right) = \frac{1}{N}\left(\frac{dK}{dt} - x\frac{dN}{dt}\right) = 0$$

Denoting (dK/dt) by $I = S = sY$, and (dN/dt) ($1/N$) by (n):

$$\frac{dx}{dt} = sy - nx$$

The equilibrium value (x^e) of (x) is then (sy/n). If $sy > nx$ for $x < x^e$ and $sy < nx$ for $x > x^e$, this growth path is stable. If the directions of these inequalities are reversed, the path is unstable. There may also exist no positive value for (x^e), or there may be multiple values, alternating between stability and instability.

of additional investment exceeds its output-increasing effect. As a result, increased investment more than justifies itself economically, and the economic system operates to increase investment *still further*. (The range of instability has an upper bound, of course, at the economy's capacity to produce, and a lower bound determined by the condition that gross investment (net of depreciation) cannot be negative. Both these boundaries or "buffers" increase over time as the economy grows.)[7]

Similar arguments follow for a downward deviation from (3), or for a departure in either direction from any other Harrod-Domar exponential growth path of income or investment.

Return to Japan

To any two economies, Country 1 (Japan) and Country 2 (the rest of the world) let us apply equation (4), the "income" version of the Domar model:

$$Y_1 = Y_{01}e^{\alpha_1\sigma_1 t} \text{ and } Y_2 = Y_{02}e^{\alpha_2\sigma_2 t}$$

$$\frac{dY_1/dt}{Y_1} = G_1 = \alpha_1\sigma_1 \text{ and } \frac{dY_2/dt}{Y_2} = G_2 = \alpha_2\sigma_2$$

Strictly speaking, we should write each (α_i) or (σ_i) coefficient as $(\alpha_i(t))$ or $(\sigma_i(t))$, since these coefficients vary over time.

If the Japanese gowth rate (G_1) exceeds the "outside" growth rate (G_2), the "equilibrium" explanation is that the Japanese propensity to save (α_1)[8] exceeds the "outside" propensity to save (α_2), and/or that the productivety of Japanese capital, both direct and indirect (σ_1) exceeds the productivity of capital elsewhere (σ_2).[9] The "disequilibrium" explanation, while not denying either of these, adds, as an additonal possibility, the hypothesis that (d_1), defined as $(G_1-\alpha_1\sigma_1)$ generally exceeds (d_2), defined as $(G_2-\alpha_2\sigma_2)$ in algebraic value. (Both deviation terms (d_i) may be negative.)

Japanese Intervention

Our equations (1–4) include the public as well as the private sector. Public investment, presumably defined consistently over time, is included in (I). Income generated by all public expenditure — usually defined as the public payroll — is included in (Y).

Japanese Goverment intervention in the economic development process can be interpreted generally as attempts to increase $(\alpha_1, \sigma_1$, or $d_1)$, either directly through its own activities or

7) J. R. Hicks has developed this combination of instabilities, ceilings, and floors into *A Contribution to the Theory of the Trade Cycle* (Oxford: Oxford University Press, 1950). A more elementary representation is to be found in: R. C. O. Matthews, *The Trade Cycle* (Cambridge: Cambridge University Press, 1959), ch. ii.

8) In most elementary growth models, (α)-often written (s)-may serve equally well for the marginal and the average propensity to save, since the divergence between the two values is ignored.

9) This hypothesis is often stated in terms of its reciprocal; the capital coefficient in Japan is supposedly lower than the corresponding coefficient elsewhere.

indirectly through influencing, planning, of "guiding" the activities of the private sector. (This is not to deny instances of special military or "favoritistic" purposes.) Since the Government is an integral part of the economy, however, stressing its role need not require "disequilibrium" analysis. To put the matter differently, the Japanese Government helps determine the values of (α_1 and σ_1), as well as inducing deviations (d_1) between the actual growth rate and the equilibuim rate (G_1), which equals the product ($\alpha_1\sigma_1$).

Exmalpes of fairly constant Japanese Government pressure for a high saving ratio (α_1) are provided by willingness to balance its own budget and to finance itself by regressive taxation on relatively high-consumption classes. One might also cite the favorable tax treatment of bank interest, of stock dividends, and of capital gains. More important, however, I should consider the generally inflationary, but never hyper-inflationary, trend of Japanese finance, dominated by the Government-controlled Bank of Japan. This imposes "forced frugality" — a better term than the conventional "forced saving" — upon the population in two ways. It prevents the public from engaging in as much real consumption in most periods as they had wished or anticipated at the prices prevailing at the outset of the period. At the same time, the credits extended go initially to business firms and other investors. Being "persons of first impact," they have the best opportunity to beat the price rises with their investment-type purchases, and influence the expenditure pattern in the "investment" direction.

As an example of the constant Japanese pressure to raise or maintain the measured productivity of physical capital (σ_1), one might cite the lag in the construction of public housing, public schools, flood control systems, and public works quite generally, so that long-term bank credit may be concentrated upon the construction of factories, the installation of machinery, and the expansion of inventories. Another laggard has been a "modern" social security system, although health insurance has run ahead of the United States. Economists who take the "human capital" notion seriously fault the Japanese preferences for factories and machinery as poor strategy for the longer run — as compared particularly to houses and schools.[10] Whatever future generations may conclude, the present Japanese strategy does wonders for the contemporaneous behavior of (σ_1).

The propensity to save, (α_1) may, however, soon fall with "high mass consumption," if indeed it has not done so already. The productivity of capital, (σ_1), may also start to fall, if it has not done so already, when Japan catches up with Western industrial technology and must direct its own research and development investment, and when the people's demand for low-productivity housing, education, public works, and welfare-state institutions must be assuaged. If the Japanese growth rate (G_1) is to continue sprinting ahead of the foreign rate (G_2) despite declines in ($\alpha_1\sigma_1$), this will be due increasingly to the disequilibrium deviation (d_1). It is this term which I have previously associated with "private pyramid-building." Let us consider how it operates.

10) An unpublished example, which crosses my desk as these paragraphs are being written (March, 1969) is K. K. Kurihara, *The Growth Potential of the Japanese Economy* (mimeographed, Binghamton, N. Y.), ch. viii.

We have, for Japanese investment expenditure in disequilibrium, as per the Domar model:

$$G_1=\frac{dI_1}{I_1}=\alpha_1\sigma_1 dt+d_1$$

This is presumed to be not only a disequilibrium but an unstable situation. This means that the deviation (d_1) can be manipulated and kept positive with greater ease, by private and public planners, than might be the case if natural, automatic, or market forces were tending to reduce its value to zero.

As $(\alpha_1\sigma_1)$ declines, (d_1) must be positive and rising, if growth is to accelerate, or even continue at its previous rate. How may (d_1) be raised? Public or semi-public investment is, in many countries, a favored route, but this route has not characterized Japan, and often operates to reduce (σ) and (G) in the longer term. The more characteristic method of Japan's *yūdō keizai* or guided economy appears to be inducement of the private sector to maintain the increase rate of private investment (dI/I) on its own, largely dependent of both the productivity of the investment and the demand for its final products.

Differential access to bank finance at low (disequilibrium) interest rates is a favored device in Japan. It is achieved indirectly by tolerance, and sometimes actual encouragement, of *zaibatsu*-type linkages between the large City Banks and their ostensible industrial debtors. A related method is the restriction of competition, through the licensing of investments by independent and uncontrollable foreign firms,[11] and the fostering of cartel arrangements among domestic ones. A third method is the reduction of risk from excess capacity, i. e., from building capacity far ahead of demand — a matter of markedly less concern to Japanese than to Western business.[12] This characteristically modern-Japanese unconcern with the overhang of excess capacity is due, I have argued, both to the high expected growth rates of Japanese income and exports and to the implied promise that firms will be awarded market shares based on capacity in any recession. Such an implied promise transforms the profitability of caution into the profitability of rashness, even when the cautious man is the better forecaster — so long as his rashness stays within the limits of *yūdō-keizai*. The Japanese "private pyramids" are composed of the resulting reserve facilities, valued for the purchasing power generated by their construction rather than for the (unused) productive power which they represent.

Profitability Considerations

Even with the guidance of an indicative plan, and its implied insurance against the recessionary consequences of over-expansion, why should firms continue building private pyramids in the absence of profit? Profit accrues, we can agree, from facilities being used, not from facilities

11) For a good summary of Japanese restrictions in this area, see W. W. Lockwood, "Poltical Economy,'" in Herbert Passin (ed.), *The United States and Japan* (Englewood Cliffs; Prentice-Hall, 1966), pp. 124–126.

12) Compare particularly Kurihara, *op. cit.*, p. 156.

held idle for possible employment in an uncertain future. Part of the answer, but only part, is the prospect of future profit if recession does not occur, a prospect enhanced by the high money cost of postponing expansion in an inflationary milieu, and the low interest cost to firms borrowing from their own banking affiliates. ("We owe it to ourselves," in the private sector.) The other part may be an improving relationship between gross profits (non-labor income)[13] and the capital productivity (σ) of our equations.

Let gross profit be $\left(K\frac{\partial F}{\partial K}\right)$, while, as per note 4, above:

$$\sigma=\frac{\partial F}{\partial K}+\left(\frac{\partial F}{\partial N}\frac{dN}{dK}\right)$$

If the marginal-productivity theory of input demand holds approximately, the incremental return from an investment is divided, with $(\partial F/\partial K)$ going to gross profit and $\left(\frac{\partial F}{\partial N}\frac{dN}{dK}\right)$ going to labor. It follows that any reduction in $\left(\frac{\partial F}{\partial N}\frac{dN}{dK}\right)$ permits the maintenance of the profit rate and total profits, despite a downward tendency of (σ). There is evidence that something of the sort is coming about, despite the rise in wage rates, by reason of the shift to labor-saving machinery, which lowers (dN/dK) and may even render it negative in some sectors of the economy. We may also find wage rates lagging behind the marginal productivity of labor despite their upward trend.

Our hypothesis, then, is that automation-cum-exploitation are significantly responsible for maintaining Japanese profits and encouraging Japanese investment to increase, despite the downward pressure of excess capacity upon measured capital productivity in Japan. Our evidence for this proposition is not conclusive. It arises from the failure of the Japanese labor share to rise, contrary to its behavior in most Western countries.

If the production function $Y=F(K, N)$ were linear homogeneous of the Cobb-Douglas type, the labor share under free competition would be constant.[14] In most countries, the labor share tends actually to rise. This tendency is ascribed by "theorists" mainly to a low (and declining)

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13) This concept of gross profit is similar to that of the English classical school. It differs from the Marxian "surplus value" by excluding payments to managerial, supervisory, and other forms of indirect labor.

14) For a production function of the Cobb-Douglas type, linear in the logarithms of the variables:

$$Y=Y_0K^{\beta}N^{1-\beta}$$

This form fits aggregate data for a number of countries, both over time and for a "cross-section" of sectors or industries. It is linear homogeneous because $F(\lambda K, \lambda N)=\lambda Y$, where ($\lambda$) is any positive constant.

Differentiating with respect to (K), we have, for the competitive rate of return (marginal product of capital):

$$\frac{\partial Y}{\partial K}=\beta Y_0K^{\beta-1}N^{1-\beta}$$

and for the profit share in total product:

$$\frac{\partial Y}{\partial K}\frac{K}{Y}=\frac{\beta Y_0K^{\beta}N^{1-\beta}}{Y_0K^{\beta}N^{1-\beta}}=\beta$$

A similar argument yields $(1-\beta)$ as the competitive labor share, with the two shares adding to unity.

In the case of the Cobb-Douglas function, the elasticity of substitution is unitary regardless of the values of the statistically-fitted coefficients (Y, β). By the Hicks formula, as developed in his *Theory of Wages* (London: Macmillan, 1932), p. 245, the elasticity of substitution equals:

substitutability of capital for labor, as the capital stock increases and labor becomes the "strategic" input in production.[15] ("Institutionalists" prefer to explain the rising labor share by the rising importance of trade unionism, and on the enactment of pro-labor legislation.) Japan, however, is an exception to our generalization of a rising labor share. We ascribe the same exceptional behavior to the activities of "mighty MITI" in maintaining prices and profit margins above their competitive levels as a necessary *quid pro quo* for accelerated expansion.

Let us consider the Japanese distribution statistics in somewhat more detail. Reliable series covering long periods are unavailable. A recent (and still unpublished) series by Kazushi Ohkawa and Henry Rosovsky covers the period subsequent to the Russo-Japanese War, and the entire Japanese economy exclusive of agriculture.[16] Ohkawa and Rosovsky estimate the growth rate of the labor share indirectly, by adding the computed growth rates of the labor force and the average wage, and subtracting the growth rate of income.[17] The figures are all negative (with a single exception) for periods delimited by wars and business-cycle extreme points, and the negative figures are particularly large in boom periods. The *final* period (three years, 1961–1963 inclusive) shows an increase in the labor share, where our theory would have suggested a decline, but the earlier postwar period (1955–1961) fits our hypothesis particularly well.

The Ohkawa-Rosovsky series is reproduced below, in preliminary form:

Period	Labor Share Growth Rate
1905–1919	−0.56 points.
1922–1931	−0.99 〃
1931–1937	−1.57 〃
1955–1961	−2.88 〃
1961–1963	0.58 〃

Another study, keyed more closely to economic theory than to economic history, includes

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$$\left(\frac{\partial Y}{\partial K}-\frac{\partial Y}{\partial N}\right)\Big/ Y\frac{\partial^2 Y}{\partial K\partial N}$$

In the Cobb-Douglas case, with $Y=Y_0K^\beta N^{1-\beta}$:

$$\frac{\partial Y}{\partial K}=\beta Y_0K^{\beta-1}N^{1-\beta},\quad \frac{\partial Y}{\partial N}=(1-\beta)\,Y_0K^\beta N^{-\beta}$$

$$\frac{\partial^2 Y}{\partial K\partial N}=\beta(1-\beta)\,K^{\beta-1}N^{-\beta}$$

so that the elasitcity of substitution is unitary.

15) Constancy of a share implies a unitary "elasticity of substitution," under competitive conditions. Fractional values imply rising shares for the inputs which increase least rapidly, and values in excess of unity imply rising shares for the inputs which increase most rapidly.

16) Ohkawa and Rosovsky, "Twentieth Century Japan: The Economics of Trend Acceleration" (mimeographed; Cambridge, Mass.: Harvard University, 1969). The five periods considered are 1905–1919, 1922–1931, 1931–1937, 1955–1961, and 1961–1963.

17) If (N) is employment and (w) the wage rate, the growth rate of the labor share is:

$$\frac{d}{dt}\left(\frac{wN}{Y}\right)\Big/\frac{wN}{Y}=\frac{dw/dt}{w}+\frac{dN/dt}{N}-\frac{dY/dt}{Y}$$

annual data for the capital share and the elasticity of substitution for the entire period 1930–1960, including even the war years 1941–1945, for the Japanese private sector, again excluding agriculture. It is due to Professor Ryuzo Sato, and shows the capital share rising every year over the interval 1947–1960, but still below its values in the 1930's.[18] Its peak value (.400) was reached in 1927; its trough (.019) came in 1947; its latest value, for 1960, was .252. Sato's estimates of the elasticity of substitution are above unity for the pre-war period 1930–1941, with a peak (1.29) in 1934. Postwar estimates are far below unity, but rising with a trough (0.16) in 1949 and a peak (0.45) in 1958. Recent values cluster about 0.40, which, Sato tells us, is close to the U. S. value. Sato's results (rising capital share, fractional elasticity of substitution) are also consistent with our hypothesis. Unfortunately, they do not extend beyond 1960, to the period where the Ohkawa-Rosovsky series indicates a rising labor share.

Conclusion

We cannot claim to have proved with any rigor our claim that Japanese growth is basically a disequilibrium phenomenon, marred by "engineered" excess capacity, questionable social priorities, and exploitation of labor and consumers in the neo-classical as well as the Marxist sense of the term. In particular, the distributional peculiarities of the Japanese economy may have non-exploitative explanations: the Cobb-Douglas function may not fit Japanese data,[19] and MITI may be less mighty than we believe it is, relative to similar public and private bodies in other capitalist countries. Nevertheless, this sort of case is worth more consideration than it usually receives either in Japan or America, either among the complacent or the suspicious, as part of the explanation for the alleged "mystery" or "miracle" of Japanese economic growth.

18) Sato, "Technical Progress and the Aggregate Production Function of Japan (1930–1960), *Riron-Keizaigaku* (March, 1968), Table 1, p. 17; also p. 22.

19) Sato (*ibid.*, Table 2, p. 20) has fitted a Cobb-Douglas function to Japanese data for 1930–1960. He obtains a surprisingly good fit, with a correlation coefficient of 0.9755, considering that his estimates of elasticity of substitution vary so widely from the unitary value assumed by the Cobb-Douglas function (note 14, above). On the other hand, he obtains a negative value for the (Y_0) coefficient, which is difficult to reconcile with "economic common sense." Sato's Cobb-Douglas function is:

$$Y=(-.7089)\,K^{.3585}N^{.6315}$$

and its capital-share coefficient (.3685) is inconsistent with all Sato's post-1945 data.