ON JAPAN'S HOUSEHOLD UTILITY FUNCTION AND CONSUMER DEMAND

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I. *Introduction*

To represent the largest component of GNP, consumer demand functions assume an importance in econometric models of an economy. To be theoretically acceptable, they have to be derived from utility functions of households. In reverse, utility functions can be estimated from observed consumer behavior. This approach has seen increasingly wider applications in recent years especially after the proposal of additive utility functions by Frisch (1959) and Houthakker (1960) a decade ago, which led to a drastic simplification of consumer demand functions. However, this approach does not seem to have had a wide acceptance in Japan. The exception is Professor Tsujimura's work (1964, 1968) based on a special type of an additive utility function.

This paper is intended to introduce (in Section II) the system of consumer demand functions associated with another special type of additive utility functions that leave the own-price elasticities of demand constant and to discuss (in Section III) its exploratory application to Japan's urban households. In so doing, it becomes necessary to review critically the alternate demand sytem proposed by Tsujimura. This is presented in Section IV.

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II. *Additive utility functions and consumer demand systems*[1]

An additive utility function is given by

$$(2.1)\quad u(x_1, \cdots, x_n) = u_1(x_1) + \cdots + u_n(x_n),$$

where $x_1, \cdots, x_n$ are the amounts of n goods consumed by a household. We assume $u_i' > 0$, $u_i'' < 0$[2]. $p_1, \cdots, p_n$ are the prices of the n goods and $I = \sum p_i x_i$ is the consumer's total expenditure or income for short. From the maximization of the utility level subject to the budget constraint, we can derive a system of demand functions

$$(2.2)\quad d \log x_i = \eta_i d \log I/p_a - \sigma\eta_i d \log p_i/p_m$$

in incremental form. p_a and p_m are two price indices defined by

$$d \log p_a = \sum \theta_i d \log p_i,\ d \log p_m = \sum \mu_i d \log p_i$$

where θ and μ are the average and marginal expenditure shares, i. e.

$$\theta_i = p_i x_i / I \text{ and } \mu_i = p_i \partial x_i / \partial I.$$

η_i is the income elasticity of demand for good i, $\dfrac{I}{x_i}\dfrac{\partial x_i}{\partial I}$.

(2. 2), thus, shows that the demand for good i is broken down into the real income effect and the relative price effect. We may regard $\sigma\eta_i$ as the own-price elasticity of demand for good i. All additive utility functions share a remarkable property that own-price elasticities of demand stand in a fixed proportion to income elasticities. Consumer demand functions are specified by n income elasticities and the common factor σ. As $\sum \eta_i \theta_i \equiv 1$, the total number of parameters to be estima-

1) The present section summarizes Sato (1969).

2) u_i need not be positive.

ted is n.

Nothing, however, is said about the constancy of the parameters of (2. 2). Suppose all $\sigma\eta_i$ are constant. Correspondingly, a specific form must be given to (2. 1). Let $\sigma_i=\sigma\eta_i$. Then, (2. 1) is represented by

(2. 3) $$u(x)=\sum c_i x_i^{-\rho_i},\quad \rho_i=\frac{1-\sigma_i}{\sigma_i},\quad c_i\rho_i<0^{3)}.$$

This is a generalized CES function familiar in production function analysis. We can consider σ_i as the average elasticity of substitution of good i for all goods. As we readily get

$$\sum \sigma_i\theta_i=\sigma\sum \eta_i\theta_i=\sigma,$$

σ is the overall average elasticity of substitution[4].

The usual practice in estimating (2. 2) has been to assume the constancy of all η' s and σ. Since η' s cannot remain constant unless they are all unity, this assumption must be considered as an approximation. The closest we can go to justify it is to assume the constancy of all $\sigma\eta_i$. When σ is relatively insensitive to changes in real income, we can assume that η' s and σ are roughly constant during the sample period under study[5].

By introducing autonomous demand shifts (β_i) and changes in family size (m) as additional terms, we may augment (2. 2) to

(2. 4) $$d\log x_i=\eta_i d\log I/p_a-\sigma\eta_i d\log p_i/p_m +\gamma_i d\log m+\beta_i,$$

which can be estimated from time-series data. These shifts in damand correspond to changes in c's in (2. 3). Let $c_i(t)=\bar{c}_i \exp c_i't$ where $\bar{c}_i$ is the means over the sample period. Then,

$$\bar{c}_i=\left(\frac{\bar{\theta}_i}{\rho_i\bar{x}_i^{-\rho_i}}\right)\Big/\left(\sum\frac{\bar{\theta}_k}{\rho_k\bar{x}_k^{-\rho_k}}\right)$$ ($\bar{\theta}$ and $\bar{x}$ are sample means)

and

$$c_i'=\frac{\beta_i}{\sigma_i}+\frac{\gamma_i}{\sigma_i}\frac{\dot{m}}{m}-\sum c_k\left(\frac{\beta_k}{\sigma_k}+\frac{\gamma_k}{\sigma_k}\frac{\dot{m}}{m}\right).$$

We call these demand shifts neutral in the sense that they leave all σ_i unchanged. This is analogous to the definition of neutrality in technical change.

III. *Application to Japanese urban households*

The results of a full-scale application of (2. 4) and its more advanced variants[6] will be reported later. In this section, we limit ourselves to exploratory discussions.

(a) *Cross-section and time-series estimates of income elasticities.*

If demand shifts are neutral, we expect that income and price elasticities of demand remain stable (if not strictly constant) over time. The stability of income elasticities can be tested by examining changes in cross-sectionally estimated income elasticities. If there are pronounced trends or other regularities in these changes, we must conclude that demand shifts have not been neutral. The utility function itself has undergone non-neutral changes.

If cross-sectional income elasticities are stable, they should also be applicable to time series. Then, the only other parameter of primary importance is σ. If all η_i are known, it is not difficult to estimate σ.

A few years ago, the Economic Planning Agency prepared a medium-term plan, in which a large-scale econometric model was used for the first time in Japanese economic planning. Con-

3) when $\sigma_i=1$, replace the i th term by $c_i \log x_i$, $c_i>0$.

4) Though $1/\sigma$ is equal to the income elasticity of the marginal utility of income, Frisch was wrong in giving it cardinal meaning.

5) As shown in Sato (1969), the elasticity of σ with respect to real income is less than 0. 2. If one wish, one can make (2. 2) more exact by taking into account the variation of σ.

6) I have in mind a system of consumer demand functions associated with a utility function additive on multi levels.

sumer demand functions were estimated (under the supevrision of Tsujimura) for sixteen expenditure categories for urban households. The sample period was 1951–1962[7]. Those functions are in a conventional double-log form

(3. 1) $\log x_i = \alpha_{0i} + \alpha_{1i} \log I/p_a + \alpha_{2i} \log p_i/p_a.$

Table 1 contains the estimates of the income and price elasticities.

We can compare these time-series estimates with cross-section estimates, which were prepared by the Prime Minister's Office (1964) for urban workers' households. The simple averages for 1953–1962[8] are shown in table 1. The two sets of estimates were prepared independently. But we can note the striking similarity between them with a simple correlation coefficient of 0.86. However, as no restirction was placed on the price elasticities in estimating (3. 1), their estimates are not necessarily reasonable. Thus, (2. 2) or (2. 4) strongly recommends itself in place of (3. 1).

It is useful to have a good initial estimate of σ in estimating our demand functions. This is provided by the price elasticity estimates of (3. 1). It can be argued that the ratios of α_{2i} to α_{1i} should center around $-\sigma$. The median of these ratios is found to be -0.687. Studies in other countries yielded estimates of σ between 0.3 and 0.7 so that this estimate is not unreasonable though it may be on a high side. $\sigma = 1/2$ is a rule-of-thumb value to start with.

(b) *Demand shifts.*

We can easily construct the additive utility function (2. 3) numerically at a given point of time from our cross-section estimates of income elasticities, the estimate of σ, and the average expenditure shares. The associated system of consumer demand functions (2. 2) is also derived. If we apply the latter to time-series data, we can estimate the demand shifts as residuals (the family size effect is not separated). Such estimates can be used as an independent check since we can compare the results with our qualitative information of consumer behavior. This approach yields table 2. Changes in demand from 1953 to 1962 and 1962 to 1966 are broken down into three components——the real income effect, the relative price

Table 1. Time-series and cross-section estimates of demand elasticities

Expenditure category[a]	time-series[b] income	time-series[b] price	cross-section[c] income
1	.24	——	.31
2	.63[d]	−1.11	.61
3	.94	−0.10	.79
4	.73	−1.04	.44
5	.86	−0.71	.70
6	.61[e]	−1.48[e]	1.10
7	1.27	−0.31	1.37
8	1.29	——	.79
9	1.08	−0.59	.57
10	.93	——	.77
11	1.60[d]	−3.52	1.48
12	1.06	−0.73	.67
13	1.51	−0.75	1.28
14	1.27	−0.69	1.31
15	.37	——	.09
16	1.65	——	1.79

Source : Economic Planning Agency (1965) ; Prime Minister's Office (1964).

a) In all subsequent tables, expenditure categoiries are numbered as follows:

1 cereals
2 fresh and dried vegetables and seaweed
3 meat, fish and dairy products
4 processed food
5 cakes, fruits, beverages, etc.
6 food prepared outside households and other food
7 clothing
8 fuel and light
9 water charges
10 rent
11 furniture and household equipment
12 personal care and health expenses
13 transport and communications
14 recreation and entertainment
15 tobacco
16 education and miscellaneous expenses

b) Economic Planning Agency (1965), p. 112.

c) Prime Minister's Office (1964). Arithmetic averages, 1953–1962 (January to November).

d) Extraneous estimates.

e) From the follow-up study (1967), p. 63. The original estimates were 2.22 (income) and −6.37 (price), which are unreasonable.

7) There are two follow-up studies of the model. See Economic Deliberation Council (1967, 1968).

8) 1951 and 1952 are excluded because of their significant difference from subsequent years.

effect, and the residual, considered as neutral demand shifts, using the cross-section estimates of income elasticities in table 1 and $\sigma=1/2$.

Table 2 shows that the real income effect is the most influential among the three components. The relative price effect, with a few exceptions, is relatively small. On the other hand, the demand shifts are very important. From 1953 to 1962, demand shifts were significantly negative for cereals (1) and vegetables (2) and significantly positive for food outside households (6), water charges (9), rent (10), furniture and household equipment (11), personal care and health expenses (12), and transport and communications (13). This pattern confirms the well-known fact that traditional diet lost consumers' favor, while housing and consumer durables gained it during the period of the 1950's (though part of the shifts must be attributed to the shrinkage of average family size). The demand shifts were equally important from 1962 to 1966, but there seems to have been a reversal in trends. E.g., food outside households (6), rent (10), and furniture and household equipment (11) now exhibited negative shifts[9]. However, the persistence of negative demand shifts for cereals (1) and vegetables (2) must be noted. This must be due to the increased preference to Western-type food.

What hypothesis can explain these patterns of change? The experience of the 1950's may be explained by the after-effect of the war period. Consumption patterns were heavily distorted in the wartime economy and postwar devastations. The living standard steadily worsened from 1934–36 to the end of World War II and further plunged downto about 50% of the prewar peak at the end of the war. The level of living of 1934–36 was recovered only as late as 1954. Per capita real consumption stood at 100 at these two time points. There is an interesting contrast bewteen these time points as demonstrated in table 3. We can note a surprising similarity of cross-section income elasticities as far as these broad expenditure categories are concerned. If we apply our

Table 2. Changes in demand : urban households, 1953–1962 and 1962–1966

Expenditure category	1953 to1962					1962 to 1966				
	x_i	I/p_a	p_i/p_m	shifts	annual shifts	x_i	I/p_a	p_i/p_m	shifts	annual shifts
1	−.175	.121	0.20	−.279	−.037	−.119	.041	−.015	−.141	−.034
2	−.177	.252	−.118	−.255	−.033	−.022	.083	−.014	−.085	−.021
3	.413	.338	−.031	.090	.010	.121	.109	−.233	.037	.009
4	.153	.176	−.023	.003	.000	.001	.059	−.009	−.049	−.012
5	.446	.294	.029	.086	.009	.238	.096	.025	.111	.027
6	1.442	.500	−.009	.643	.057	.088	.155	−.020	−.039	−.010
7	.734	.657	.139	−.081	−.009	.051	.197	.034	−.159	−.038
8	.332	.338	.011	−.015	−.002	.273	.109	.074	.070	.017
9	.410	.234	−.039	.189	.020	.212	.078	−.002	.126	.030
10	.560	.328	−.180	.433	.041	.005	.106	−0.43	−.050	−.012
11	3.335	.725	.180	1.139	.088	.322	.214	.117	−.027	−.007
12	.580	.280	.018	.213	.022	.212	.092	.014	.094	.023
13	.883	.603	−.022	.201	.020	.308	.183	.009	.116	.028
14	.485	.620	−.130	.054	.006	.074	.187	.094	−.002	.000
15	.087	.034	.010	.041	.005	.145	.012	.010	·121	.029
16	.746	.934	.022	−.077	−.009	.367	.264	−.022	.103	.025

Source: Basic data on income, expenditures, and prices for 1953–1962 are from Tsujimura (1968), table 15-1. For 1962–1966 Prime Minister's Office, *Annual Reports of the Family Budget Survey* are the source of data.

a) Changes are measured by the ratios of terminal to initial values minus one.

9) We get −0.76 as the simple coefficient of correlation between the annual rate of demand shifts for the first subperiod and its difference from that of the second subperiod.

utility function, we can interprete this to mean that demand shifts were neutral between 1934–36 and 1954. (Note the independence of income elasticities from the relative price structure.) Expenditure shares increased for food and decreased for housing. Clothing, fuel and light, and miscellaneous category maintained constant shares. As the real income effect is zero, demand shifts can be derived by eliminating the relative price effect. Table 3 gives these hypothetical shares for 1954. Clearly, consumer demand shifted toward increased consumption of food and reduced consumption of housing services. Then, the demand shifts from 1953 to 1962 largely reversed this trend. They may be considered as the return to what should have been a normal pattern of consumption. In other words, there might have been a normal utility function toward which the consumers moved.

This does not imply that there were not other new factors that influenced changes in consumer tastes. For instance, continued urbanization must have had a significant influence on molding consumption patterns. A more intensive investigation is necessary to formulate an adequate hypothesis to explain the consumer behavior of the 1950's and the 1960's.

(c) *The demand for new commodities, especially consumer durables.*

Shifts in demand are due either to changes in tastes for existing goods or to the introduction of new commodities that create new tastes. Postwar years brought Japanese consumers into abrupt and direct contacts with Western mores and habits. However, their influences must have been only gradual as Ohkawa and Rosovsky (1961) demonstrated that indigenous goods still occupied a major proportion of consumer expenditure in the mid-1950's. But toward the end of the 50's, a wave of new consumer durables swarmed into the market, demand for which expanded at explosive rates. They are mostly electric appliances including electric fans, electric washing machines, refrigerators, vacuum cleaners, electric rice cookers, transistor radios, tape recorders, stereophonic sets, and TVs. Within a decade, these durables were acquired by the majority of households and turned themselves from luxuries into necessities.

When such drastic changes in demand take place, our assumption of stable income elasticities is likely to break down for these new goods because we can no longer work with the representative household, which underlies our use of a single utility function to represent all households[10].

Table 3. Average expenditure shares, income elasticities, and relative prices, urban worker households, 1934/36 and 1954

Expenditure category a)		food	clothing	fuel and light	housing	miscellaneous	total
Average expnditure shares	1934/36	.358	.115	.049	.166	.310	1.000
	1954 actual	.455	.126	.050	.058	.311	1.000
	hypothetical b)	.480	.154	.035	.051	.280	1.000
Income elasticities	1934/36	.546	1.346	.626	.948	1.491	1.000
	1954	.585	1.383	.697	.922	1.527	1.000
Relative prices	1934/36	1.000	1.000	1.000	1.000	1.000	1.000
	1954	1.201	1.311	.736	.436	.878	1.000

Source: Prime Minister's Office(1964). 1934/36 refers to Sept. 1934 to August 1937.

a) The correspondence to table 1 is as follows:
food(1 to 6), clothing(7), fuel and light(8), housing(9 to 11), miscellaneous (12 to 16).

b) Obtained by dividing the actual shares by the relative price effect.

10) Because of their indivisibility and durability, the demand for consumer durables requires ordinarily separate treatment from nondurables.

When a new good (especially a durable) is introduced, the demand for it typically follows a logistic curve of growth. With a new consumer durable which is relatively expensive, it is purchased initially by high-income households. At this point, the demand has a very high cross-section income elasticity. Through the demonstration effect and intensive advertising efforts, the demand spreads to medium-income households. The income elasticity falls while demand grows at a brisk rate. The consequent expansion of production leads usually to lowered prices, which bring the durable within the reach of lower-income households. The last stage of the logistic curve is reached. The income elasticity may fall even below unity because high-income households are already satiated with the good. From then on, the demand for the durable is determined by replacements and new entry of households.

Among our sixteen expenditure categories, furniture and household equipment (11) contain consumer durables——notably electric appliances and vehicles. From the late 50's to the early 60's a large number of new durables were introduced. Table 4 shows the rates of acquisition of major appliances and passenger cars by households. We can see that the phenomenal growth of TV ownership was confined in a short span of five years from 1958 to 1963. Other appliances registered less spectacular but still impressive records of growth at about the same time.

Turning to table 5, which shows the cross-section income elasticity of demand for furniture and household equipment as well as percentage shares of these major appliances and vehicles in the total expenditure on this category, we can see the income elasticity (after smoothing out random fluctuations) reached a peak of 1.9 at 1959. The percentage share also reached its peak. From 1959, the elasticity steadily decreased and went down below unity by 1964. By the mid-1960's, consumer demand was satiated with these light durables that had been introduced in the late 50's. What are now known as the 3Cs-color TVs, coolers (air conditioners), and passenger cars-were still very expensive and the take-off in demand growth was still in the offing. Thus, the income elasticity of demand remaind low since 1964.

This example illustrates the danger of a mechanical application of a simple-minded model to explain changes in tastes and their effect on consumer demand. However, it should be emphasized that this does not mean that we must abandon our utility function analysis. The advantage of an additive utility function lies in enabling us to separate these commodities that require special treatment from the rest of goods and services which can be dealt with by a mechanical analysis.

Table 4. Percentage ownership of major electrical appliances and vehicles, 1957–1967[a)]

year/month	TVs	color TVs	electric washing machines	refrigerators	electric fans	air conditioners	electric rice cookers	passenger cars
1957/9	7.8	…	20.2	2.8	21.6	…	…	…
1958/2	10.4	…	24.6	3.2	22.6	…	9.0	…
1959/2	23.6	…	33.0	5.7	28.6	…	20.7	…
1960/2	44.7	…	40.6	10.1	34.4	0.2[b)]	31.0	1.2[b)]
1961/2	62.5	…	50.2	17.2	41.9	0.4	41.8	2.8
1962/2	79.4	…	58.1	28.0	50.6	0.7	48.4	5.1
1963/2	88.7	…	66.4	39.1	60.6	1.3	52.9	6.1
1964/2	92.9	…	72.2	54.1	67.4	1.7	55.7	6.0
1965/2	90.3	…	72.7	62.4	70.7	2.0	57.6	9.1
1966/2	94.1	0.3	75.5	…	65.7	2.0	61.6	12.1
1967/2	96.2	…	79.8	69.7	69.1	2.8	…	9.5

Source : Economic Planning Agency, *Consumer Behavior Survey*.

a) Percentage of households having listed goods.

b) 1960/8.

Table 5. Percentage shares of electrical appliances and vehicles in household expenditure on furniture and household equipment (all urban households, yearly average) and income elasticities of demand for furniture and household equipment, 1953–1966,

year	percentage shares						income elasticity of demand[d)]
	electrical appliances and vehicles	electrical appliances	of which			vehicles	
			radio and TVs[a)]	electric motive appliances[b)]	other electrical appliances[c)]		
1953	…	…	…	…	…	…	1.384
1954	…	…	…	…	…	…	1.099
1955	…	…	…	…	…	…	1.895
1956	41.6	31.1	…	…	…	10.5	1.414
1957	43.1	34.5	…	…	…	8.6	1.567
1958	50.9	42.8	20.8	12.6	9.4	8.1	1.712
1959	59.5	52.2	32.8	11.9	7.5	7.3	1.876
1960	58.2	50.6	30.0	12.7	7.9	7.6	1.557
1961	56.1	48.3	23.0	18.7	6.6	7.8	1.379
1962	54.3	47.3	18.2	23.6	5.5	7.0	1.220
1963	55.5	44.3	17.1	21.8	5.4	11.2	1.180
1964	52.7	43.2	17.5	20.3	5.4	9.5	.823
1965	52.0	37.9	16.7	16.0	5.2	14.5	.772
1966	53.2	37.1	16.6	15.1	5.4	16.1	.912

Source: Prime Minister's Office (1964) and *Annual Reports of the Family Budget Survry.*
a) Including phonograph sets and tape recorders.
b) Including washing machines, electric fans, refrigerators, vacuum cleaners and blenders.
c) Including electric bulbs, fluorescent lamps, electric irons, electric cooking pans, and toasters.
d) Income elasticity of demand on the cross section of all urban worker households. 1953–63: January-November average. 1964–66: yearly average.

IV. *Tsujimura's alternate system of consumer demand*

(a) *quadratic utility function.*

Tsujimura[11)] derives his demand system from a quadratic utility function

(4. 1) $$u=\sum_{i=1}^{n}\left(a_i x_i+\frac{1}{2}a_{ii}x_i^2\right),\quad a_i>0,\quad a_{ii}<0.$$

This is an additive utility function of a special type. Therefore, consumer demand functions associated with (4. 1) are representable in the form of (2. 2). Needless to say, $\sigma\eta$'s are not constant in this system. By following the well-known procedure, we can obtain the formulas that determine η's and σ as follows:

(4. 2) $$\sigma=-\frac{1}{I}\sum\frac{p_k^2}{a_{kk}}>0$$

and

(4. 3) $$\eta_i=-\frac{1}{\sigma}\frac{p_i}{a_{ii}x_i}>0.$$

The marginal propensities to consume (MPC) are

(4. 4) $$\mu_i=p_i\frac{\partial x_i}{\partial I}=(p_i^2/a_{ii})/\sum p_k^2/a_{kk},$$

which shows that the Engel functions are linear, i.e. independent of the income level (both cross-section and time-series wise). In fact, (4. 1) belongs to a special class of additive utility functions with linear Engel functions. Both income elasticities and MPC's are very price-sensitive according to (4. 3) and (4. 4). This is a proposition that can be tested empirically.

(b) *The family size effect and the habit formation hypothesis.*

Tsujimura introduces the family size as an additional variable. To explain the demand shifts over time, he adopts the habit formation hypothesis. These are assumed to affect a_i's only in (4. 1). Thus,

(4. 5) $$a_i=\bar{a}_i+b_i m+c_i H_{it},\quad H_{it}=\sum_{v=1}^{t-1}x_{iv}.$$

11) In this section, we refer to the model and its estimates in his latest publication (1968). The model, however, is already contained in his earlier works (in particular, 1964; Tsujimura and T. Sato (1964) gives an English summary).

H_{it} is called the habit potential. Tsujimura argues that the more the consumer is exposed to a commodity through consumption, the better he can appreciate its utility and the more willing he becomes to consume that commodity. This is a "learning by doing" hypothesis on consumer behabior. Thus, $c_i>0$.

We can derive the formulas measuring the family size effect and the habit formation effect corresponding to (4. 3) and (4. 4), but they are not given here to save space. (They are dependent on relative prices, too.)

(c) *Estimates of various elasticities implicit in the Tsujimura system.*

We can now estimate various elasticities, given the estimates of the utility function parameters. We use Tsujimura's final estimates of his model ((1968), table 16-1-1), which he obtained after a tremendous amount of computational work with a great deal of ingenuity[12]. The sample period is from 1951 to 1962 and the number of goods is 16[13].

For the elasticity of substitution σ, we obtain the following values from 1951 to 1962 : 2.82, 2.51, 2.37, 2.65, 2.69, 2.64, 2.72, 2.89, 3.00, 2.97, 2.90, 2.84. These values are very high indeed in comparison with our own preliminary estimate of 1/2.

Table 6 shows the estimated income elasticities for three years 1951, 1955, and 1960. First of all, we note extreme volatility of the estimates. For some categories, income elasticity simply jumps around every time relative prices change. This is true especially with respect to those goods which we have found to have been subject to strong demand shifts like cereals (1), food prepared outside households (6), and furniture and household equipment (11). For instance, cereals are universally known for their low income elasticity of demand but table 6 gives an entirely different picture. Furthermore, these estimates exhibit very little resemblance with the time-series and cross-section estimates of table 1. (r is −0.185 with the time-series estimates and −0.0055 with the cross-section estimates).

It follows immediately that the price elasticities are about 2.4 to 3 times as large. They are also unstable. It is no wonder that Tsujimura finds that consumer demand is highly price-sensitive. This conclusion is untenable.

(d) *The role of the habit formation hypothesis.*

The factor which is most responsible for yielding these unusual estimates is the habit formation hypothesis. If its effect is overestimated, the elasticity estimates can be very much off the mark, *even if the demand system as a whole shows a good fit for the sample observations.* This point can be demonstrated by breaking down the observed changes in demand into the four effects due to changes in real income, relative prices, family

Table 6. Estimates of income elasticities of demand in the Tsujimura model—1951, 1955, 1960

Expenditure category	Income elasticities 1951	1955	1960
1	1.26	1.84	2.30
2	.22	.32	.50
3	.66	.66	.71
4	.53	.49	.44
5	.68	.60	.54
6	1.63	1.03	.73
7	2.85	.87	.71
8	.71	.61	.69
9	.42	.56	.61
10	.85	1.34	2.00
11	4.22	2.21	1.02
12	.60	.64	.59
13	.26	.41	.40
14	.60	1.02	1.18
15	.45	.46	.50
16	1.48	1.12	1.07

Source: Computed from Tsujimura (1968), tables 15-1 and 16-1-1.

12) In deriving the estimates of the complete system, he attempts direct fits of all the equilibrium conditions simultaneously including the marginal utility of income.

13) The data also underlie the medium-term plan estimates.

size, and habit potentials. Table 7 presents them for changes from 1959 to 1961 (expressed on the annual basis). The second column gives the actual changes in x_i. The third column shows the chages in x_i computed from the model, which are then broken down into the four effects. The last three of them should each add up to 0 except for rounding errors.

Because of the high price elasticities, the relative price effect is more pronounced than the real income effect (compare the absolute sums). The family size effect is as significant as the relative price effect. This is completely against our common sense because the average family size reduced only by 0.22 person from 1959 to 1961.

If we accept the Tsujimura model, we have to accept an explanation like the following : For cereals (1), the income and price effects were very strong. (Indeed, rice is a luxury good in this model). Japanese consumers would have avidly increased their consumption of rice through increased income and relatively lower prices. However, the small reduction in the average family size induced them to curtail this potential increase by one half[14]. The other half was nullified because the habit potential was not strong enough to keep rice competitive with other goods.

Another interesting example is provided by furniture and household equipment (11). Inspite of the explosive growth of demand for this category, the table shows that both the real income and habit formation effects were relatively small, the lion's share going to the price and family size effects[15]. We can contrast this with our own observation in III. (c).

There are other interesting cases, which we

14) In fact, it is computed that the family size effect is such that all demand for rice would be wiped out if the family size is reduced by 0.7 person.

15) In this model, the explosive expansion in demand for consumer durables is explained by very high income and price elasticities in the early part of the sample period (the relative price continued to decline) and by the very large family size effect. The habit formation effect is negligible.

Table 7. Decomposition of demand changes, 1959–1961, in the Tsujimura model

Expenditure category	x_i 1960	Δx_i[a)] actual	estimated	of the estimated Δx_i real income effect	relative price effect	family size effect	habit formation effect
1	3848	−130	4	422	833	−673	−578
2	1249	− 21	−109	30	−165	94	− 68
3	3167	71	− 63	107	−299	107	22
4	1134	29	5	24	− 35	− 10	26
5	3277	203	129	84	35	− 62	72
6	944	74	129	33	− 22	28	90
7	3902	391	339	132	84	293	−170
8	1608	95	15	53	− 24	− 43	29
9	153	9	9	4	1	− 5	9
10	1228	52	− 40	117	−455	238	60
11	1414	218	481	69	129	229	54
12	1675	125	135	47	58	− 25	55
13	637	52	69	12	11	13	33
14	1971	86	82	111	− 82	− 78	131
15	378	21	12	9	21	2	− 20
16	4720	180	316	241	− 87	−108	270
total	31305	1455	1513	1495	3	0	15
absolute sum	…	…	…	1495	2341	2008	1687

Source: see table 6.

a) In 1960 yen per household and per month.

leave to the reader's inspection (e. g., category 6, 7, 10, 14, 16). We can conclude that the Tsujimura model provides *ad hoc* explanations of the past events, some of which are not easily reconcilable with our common knowledge of consumer behavior at the period under study.

(e) *Simulation and prediction tests.*

The same point can be demonstrated more dramatically by means of simulation tests. We can trace the hypothetical path of consumer demand, given the estimated utility function and the time series of exogenous variables (income, family size, and prices). For instance, we may hypothetically assume that all prices are frozen at the base-year levels over the sample period. We can then compute consumer demand for all categories of goods annually. I tried this kind of an exercise by starting from the 1953 observations with the 1953 price structure prevailing through the subsequent years. It turned out that the demand for furniture and household equipment becomes negative in 1959 and, if I continue, the demand for cereals turns negative in 1960. The composition of demand by this time becomes very much different from what was actually observed.

As another test, I tried a prediction test. Starting from 1962, I took actual prices and total expenditure in current value and applied the model to predict the expenditures on the individual categories. The result is even worse than the simulation test. The demand for cereals becomes negative as early as 1965. The standard error of forecast is 0.136, 0.196, 0.563 for 1963 to 1965. These values should be compared with 0.137 found for 1966 when we predict the 1966 values from 1962 by extrapolating the demand shifts observed from 1953 to 1962 in table 2.

The observations given above force me to conclude that, in spite of its analytical novelty, the Tsujimura model is empirically untenable. However, this does not mean that the model itself is basically wrong. The awkward results we have found above can be attributed to two features of his model——the particular forms of the utility function and of the habit formation hypothesis. They can be remedied. We now discuss them below.

(f) *Form of the utility function.*

As we noted, the quadratic uitlity function belongs to a class of additive utility functions that yield linear Engel functions. It is represented by generalized CES forms of[16)]

$$(4.6) \qquad u=[\sum c_i(x_i-b_i)^{-\rho}]^{-1/\rho}, \quad \rho=\frac{1-\nu}{\nu},$$

$$\nu>0, \quad c_i>0$$

and

$$(4.7) \qquad u=[\sum c_i(b_i-x_i)^{-\rho}]^{-1/\rho}, \quad \rho=\frac{1-\nu}{\nu},$$

$$\nu<0, \quad c_i>0,$$

In the former, $\{b\}$ represents the subsistence level of consumption (if positive) below which $\{x\}$ cannot fall. If $\nu=1$, it reduces to famous Stone's linear expenditure system

$$u=\sum c_i \log(x_i-b_i).$$

In the latter, $\{b\}$ which must be positive are the saturation level of consumption beyond which $\{x\}$ does not increase. For both, the demand functions are given by

$$(4.8) \qquad p_i(x_i-b_i)=\mu_i(I-\sum p_k b_k)$$

where

$$(4.9) \qquad \mu_i=\pi_i p_i^{1-\nu}/(\sum \pi_k p_k^{1-\nu}), \quad \pi_i=c_i^{\nu}/(\sum c_k^{\nu})$$

and π_i is the base-year average expenditure share of good i. For a cross section, (4. 8) applies to all consumers without changes in MPC's. If one wants the constancy of MPC's over time, one gets the Stone system.

Tsujimura's quadratic utility function is a special case of (4. 7) with $\nu=-1$. It explains why his elasticity estimates have to be volatile. Clearly, there is no *a priori* reason why one should

16) See Pollak (1968).

pick this particular value. ν, in fact, should be determined empirically from observations. My guess is that if this class of utility functions is to be taken, (4.6) is the appropriate form. Moreover, ν is likely to be fairly small (less than 1)[17]. However, for reasons given in Sato (1969), I prefer to adopt (2.3) to this class of utility functions.

(g) *Form of the habit potentials*

Consider a stationary state in which income and prices remain constant perpetually. If consumers are following the rules set up in the Tsujimura model, their market baskets must change from year to year because the habit potentials continue to increase at different rates for different goods. Then, there will be no stationary state for consumption until the consumers buy only one good with all other goods eliminated from the baskets. This is a very unusual course of events. Tsujimura justifies it by insisting that the disequilibrating forces generated by this consumer behavior contribute to the maintenance of economic growth through inducing certain changes on the supply side. But this seems to be a far-fetched argument, especially because the whole thing can be altered by introducing a depreciation factor, i. e. by modifying the definition of the habit potential to

$$H_{it}=(1-\delta)H_{i,t-1}+x_{i,t-1}$$

This is done by Houthakker and Taylor (1966) and noted by the author himself (1964, p. 45).

17) We can get from (4. 9)

$$\frac{\Delta\mu_i}{\mu_i}=(1-\nu)\Delta\left(\frac{p_i}{p}\right)\bigg/\left(\frac{p_i}{p}\right).$$

Suppose demand shifts were neutral between 1934/36 and 1954. Then, from the data of table 3, we can obtain a regression, which gives $\nu=0.233$ with $R^2=0.5102$.

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