A Note on Capital/Output Ratio[1)]

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1. The following symbols and definitions will be used throughout this paper:

K……the existing stocks of durable producers' structures and equipment (D) plus inventories (S)

Y……the current flow of income (or output)

I……the current net addition to K, or ΔK, or $\Delta D+\Delta S$

y……the additional flow of income (or output) yielded from net investment

k……the average capital/output ratio, or K/Y

k'……the marginal capital/output ratio, or investment/output ratio, or I/y

2. What is commonly called "capital/-output ratio" nowadays has several antecedents with differing implications and applications. Sometimes it has been used in its reciprocal form and called "product/capital ratio" or "the productivity of capital." Further, it has definite affinities to such concepts as "period of production," "turn-over of capital," "organic composition of capital," and "accelerator." Without going into too refined a discussion on the subject, we may classify the use of these affiliated concepts into the following three classes:

(1) As measuring the *average life of capital goods in use*: R. W. Goldsmith, for example, who made calculations of average and marginal capital/output ratios for the United States annually over 1897–1950, says that "the actual level of the ratios is determined by the fact that in static conditions the overall ratio is equal to one-half the average life of total output."[2)] W. A. Lewis' formulation of the concept also comes to the same thing as that of Goldsmith, although his wording appears to be different from that of the latter. Lewis says: "In the mathematical sense the ratio of the existing stock of capital to income (i. e. the average as distinct from the marginal ratio) is simply a function of the proportion of national income invested, the average life of investments, and the rate of growth of income."[3)] From this, he naturally goes on to say that" given the average life of capital, the principal determinant of the capital-income ratio is the proportion of national income annually invested."[4)] This implies that, instead of writing an equation like $GC=s$, it is more appropriate to regard the rate of growth as determined by the average life of capital and the rate of investment.

(2) As measuring the *relation of incremental output to induced investment*: This is the relation visualized as proceeding from ΔY to I and most commonly indentified with the term "accelarator."[5)] Although there are

1) In the course of preparing this paper, the writer is indebted to Drs. H. W. Singer, A. K. Ghosh and Ashok Mitra for helpful suggestions.

2) International Association for Reserch in Income and Wealth, *Income and Wealth*, Series II, 1952, p. 298, footnote.

3) W. A. Lewis, *The Theory of Economic Growth*, 1955, p. 202.

4) *Ibid.*, p. 202.

many variations in this application, almost all of them use the concept in the marginal sense and conceive of I here as theoretically-induced magnitude, most typically exemplified by Harrod's C_r which is defined as "the requirement for new capital divided by the increment of output to sustain which the new capital is required."[6] The commonest use of the concept in this sense has been in the sphere of trade cycle theory and a great deal of refinements have been carried out in the past several years.

(3) As measuring *the productivity of capital*: Here again the concept is generally used in the marginal sense, but proceeding from a given I to the additional flow of net output yielded from that investment, or y according to our notation. Whereas in the use of the concept measuring the relation of incremental output to induced investment the latter is conceived as a *theoretically*-induced magnitude, the concept as used in the sense of the productivity of capital would require the additional flow of net output to be conceived as a *theoretically*-derived magnitude. That is the reason why a special notation, y, is used in this paper instead of ΔY. Theoretically, the concept in this sense has been made much use of by E. Domar in its reciprocal form, and more lately by a large number of economists who are concerned with the investment programming in under-developed economies.

The purpose of this paper is to examine the concept of capital/output ratio especially in relation to the last use mentioned, namely as a theoretical and practical instrument of investment programming.

3. In applying the concept of capital/output ratio to the investment programming of under-developed economies, there seems to be an accepted body of opinion which is best represented by a recent study by the United Nations Secretariat, "Problems and Techniques of Economic Development Planning and Programming with Special Reference to ECAFE Countries."[7] The study sets out a general proposition as follows: "The rate of economic growth may be analytically considered as being the function of two factors, (a) the rate of capital formation and (b) the captal/output ratio; accordingly, development policies may be described as aiming to increase the former, reduce the latter, or do both."[8] Here, the "capital/output ratio" is defined as "the relationship between the net capital formation of a certain time period and the additional net output that results in the first subsequent time period in which the effects of that capital formation can be fully felt,"[9] although for practical purposes it is proposed to use the ratio in the form of "the relationship between increment of capital and increment of output, both in the *same* period."[10] In other words, it is suggested that it is desirable to reduce k' (according to our notation) if we wish to speed up the growth rate of an economy and that since k' in the theoretical sense is difficult to calcu-

5) The accelerator, of course, usually relates the rate of change of consumer demand to the derived demand for investment, whereas the capital/output in this application relates, generally speaking, the rate of change of total output, ΔY, to investment.

6) R. F. Harrod, *Towards a Dynamic Economics*, 1948, p. 82.

7) *Economic Bulletin for Asia and the Far East*, Vol. VI, No. 3. November 1955.

8) *Ibid.*, pp. 25–6.

9) *Ibid.*, p. 26, footnote.

10) *Ibid.*

late an approximation might be made by computing the ratio between I and ΔY, or between investment and incremental output of the same period.

In the accepted body of opinion, to which reference has been made, the above general proposition is understood to imply that investment in industries with low capital/output ratios is preferable in those economies where the rate of capital formation is still low. In such discussions it is usually taken for granted that k' could be used in sectoral sense and further that a high sectoral k' is equivalent to a high degree of capital intensity. The statement that an under-developed economy with a large population and a low rate of saving had better emphasize, in its initial stage of economic development, investment in the so-called "quick-yielding projects" such as agricultural development rather than in "capital intensive" projects such as steel industry or ship-building is usually based upon the reasoning summarized here. Naturally, in so far as the tool of programming itself, i.e. k', either in its overall or sectoral sense, can not be isolated from various contingencies of the actual world, the proposition in question is accompanied by an array of qualifications.[11] But the essence of the matter is regarded as unimpaired by these qualifications.

4. It cannot be doubted that as a matter of abstract principle it is true to say that the rate of growth of an economy depends upon its rate of capital formation and the overall investment/output ratio. But when this proposition is applied to the investment programming of an under-developed economy, there seem to crop up a number of confusions which could be traced to a somewhat superficial use of the term "capital/output ratio." An attempt will be made, therefore, in this paragraph, to bring out the basic theoretical implications of the overall investment/output ratio, or k', with an aid of a simple model.

Let there be two industries, the investment industry (with subscript i) and the consumer goods industry (with subscript c). Assume the productivity (in the sense of the quantity of product per unit labor) of p_i and p_c, with the labor power engaged of L_i and L_c, respectively. Assume also the common wage rate of w (per unit labor) for both sectors. For simplicity's sake, it is further assumed that workers consume all of their income, that profits (Π) arise only in the consumer goods industry, and that all the profits are invested. Then:

Consumer goods produced: $p_cL_c=wL_c+\Pi$

Investment: $(p_c-w)\ L_c=wL_i$

These expressions are in units of consumer goods; but investment in units of own goods can be written: p_iL_i. If we assume the labor coefficient of α per investment good when the latter is applied in the production of

11) For example, the study by U. N. Secretariat quoted above surrounds the general proposition with such qualifications as follows: "Quick-maturing projects obviously offer relatively lower capital/output ratios from the immediate point of view, but not necessarily a lower ratio when a longer period is considered. The time element plays a key role in the concept of capital/output ratio used as an investment criterion." (*Ibid.*, p. 44) "When applying the criterion of capital/output ratios to specific investment projects or specific sectors, the supplementary benefits of any project to other economic activities should be considered, although these cannot easily be measured." (p. 44) "In a way, the lowering of the present capital/output ratio conflicts with the creation of a capital base for the economy, which is likely to involve relatively capital-intensive investment." (p. 44) And so on.

consumer goods, p_iL_i units of investment will require ap_iL_i units of labor power, and will result in the production of ap_iL_i-times-p_c units of consumer goods.[12)]

Under these simplified assumptions, the investment/output ratio, or k', can be written as follows:

$$k'=\frac{wL_i}{ap_cp_i\ L_i}=\frac{w}{ap_cp_i}\cdots\cdots\cdots\cdots\cdots\cdots(1)$$

In other words, the investment/output ratio comes to be an expression where the wage rate appears in the numerator and a combined index of productivities appears in the denominator. If the wage rate is kept constant while productivities rise, the investment/output ratio will be reduced. If the wage rate keeps pace with the rise in productivities, the investment/output ratio will not change. Such an implication is quite consistent with the proposition that given the rate of capital formation the growth-rate of an economy is larger when the investment/output ratio is smaller. But the same thing can be stated much more directly by saying that the growth-rate will be larger when the wage rate can be kept lagging behind the rise in productivity. Instead of saying that it is desirable to lower the investment/output ratio, we could point directly to factors involved and formulate the wage policy appropriate to the growth pattern desired. It might well be that the fact that the average capital/output ratio has remained almost constant for a long time in countries like the U. K. and the U. S. A. in spite of the almost certain rise in productivities is explained by the concomitant rise in the real wage rate. And again, the fact that the capital/output ratio of U. S. S. R. is said to have been quite low for some time (a little less than two according to Professor Mahalanobis) could be explained in terms of the deliberate policy of keeping wage rate constant while productivities rose. Thus, the relevance of wage policy (or its allied problems) to the development planning seems to require a greater degree of attention than given, for example, in the U. N. Secretariat study referred to earlier.

It must be noted, however, that in a free competitive economy the wage rate (w) and the productivity of labor (p_c) are not independent of each other. In other words, the numerator and the denominator of the expression (1) can be a separate object of deliberate policy only to a limited extent. But what is important is to be aware of the essential factors involved when we make use of the concept like the investment/output ratio. Usefulness of the formulation like (1) might be indicated further by relinquishing one of the simplifying assumptions, i. e., that of the common wage rate. Suppose now that the wage rate in the consumer goods industry is w_c and that in the investment goods industry w_i, and that both I and y are to be expressed in terms of w_c. Then:

$$k'=\frac{w_iL_i/w_c}{ap_cp_iL_i/w_c}=\frac{w_i}{w_c}\cdot\frac{w_c}{ap_cp_i}$$

Suppose $w_i>w_c$ and $\frac{w_i-w_c}{w_c}=\delta$, then:

$$k'=(1+\delta)\cdot\frac{w_c}{ap_cp_i}\cdots\cdots\cdots\cdots\cdots\cdots(2)$$

This expression implies that k' can be lowered by reducing δ, or the wage differential between the consumer goods industry and the investment goods industy. In actual application this point can be interpreted as sug-

12) p_c and p_i are likely to move in the opposite direction. The prgress in technology usually implies that the increase in p_c more than makes up for the reduction in p_i.

gesting that an under-developed economy which is dependent on the importation of capital goods from a high-wage economy could lower k' by learning to produce the same capital goods at home with its own labor.

5. The expression like (1) which has been arrived at on the basis of a simplified model may sound somewhat accidental. But it is by no means the case. That it expresses the essential content of the concept like capital/output ratio can be shown by adopting a different approach.

Let us start with the concept of subsistence fund in Böhm-Bawerk. In his formulation, the subsistence-fund, F, which is required to enable production in a roundabout manner extending over N years is equal to $(1+N)$ times the annual wage-bill, W, divided by two, or[13]

$$F=\frac{(1+N)W}{2} \quad \cdots\cdots\cdots\cdots (3)$$

Assumptions which underlie this formulation can be made less arbitrary in two respects, i. e., (a) the discrete stage of production which Böhm-Bawek visualized can be changed into a continuous one, that is, a stage of one year can be broken into stages of $1/n$ years with n approaching to infinity; and (b) the unilaterally moving structure of production which Böhm-Bawerk assumed can be changed into a more general one where capital goods will be assumed to be used not only for the production of consumer goods but for the production of capital goods themselves. By adopting the first of the above two modifications, we can rewrite the equation (3) into a simpler form: $F=\frac{NW}{2}$.[14] The right-hand side of this equation can be rewritten as $\frac{N}{2}.\ wL$ in which w stands for the wage rate and L for the labor employed. Now, $\frac{N}{2}.L$ in this expression is found to be equal to the total of labor power applied in all the stages of production appropriately weighted by respective period of gestation. For, in the Böhm-Bawerkian scheme, such a total should be equal to the quantity of labor power applied in each stage of production (which we may represent by a) multiplied by $(1+2+3+\cdots+N)$, or $\frac{a(1+N)N}{2}$, and when the time period is taken infinitesimally, it become $aN^2/2$ which is equivalent to $\frac{N}{2}$ times aN $(=L)$.

The expression $\frac{N}{2}.L$ (the total of labor power applied in all the stages of production appropriately weighted by respective period of gestation) will take a different form when we incorporate the second of modifications mentioned above, that is to say, when we assume that capital goods are used not only for the production of consumer goods but also for the production of capital goods themselves. The simplest manner in which we can visualize such a situation will be to assume that there is only one capital good and one consumer good, the period of production for both being one year and each of which requiring for the production of one unit c units of capital goods $(c<1)$ and a units of labor power. Under such an assumption the quantity of total labor required for the production of one unit of consumer good should be equal to $a(1+c+c^2+\cdots\cdots)$ or $\frac{a}{1-c}$, since c u-

13) For the discussion in this paragraph, the writer owes greatly to Kei Shibata, *Shin Keizai Ronri* (A New Logic of Economics), 1942, pp. 20—26.

14) First, we can rewrite the equation (3) as follows: $F=(1+nN)\frac{W}{2n}$. Then, if n approaches infinity, this becomes $F=(\frac{1}{\infty}+N)\frac{W}{2}=\frac{NW}{2}$.

nits of capital good needed for the production of one unit of consumer good require ac units of labor power and c^2 units of capital good, and these c^2 units of capital good require for their production ac^2 units of labor power and c^3 units of capital good, and so on. If we again make a summation to obtain the total of labor power applied in all the stages of production appropriately weighted by respective period of production, it comes to $a(1+2c+3c^2+4c^3+\cdots\cdots)=\frac{a}{(1-c)^2}$. This quantity multiplied by wage rate w should give us the subsistence fund required for the reproduction of one unit of consumer good. Therefore, the total subsistence fund needed for the annual output of all the consumer goods (which we shall write as Q) can be expressed as follows:

$$F=\frac{awQ}{(1-c)^2}\cdots\cdots\cdots\cdots(4)$$

Since the total wage bill required is equal to the wage rate (w) times the labor power required per consumer good ($\frac{a}{1-c}$) times the output of consumer good (Q), the roundaboutness that can be expressed by the ratio between the subsistence-fund and the total wage bill will be equal to:

$$\frac{awQ}{(1-c)^2}\div\frac{awQ}{(1-c)}=\frac{1}{1-c}$$

This should be equal to $\frac{N}{2}$ in the simpler formulation of Böhm-Bawerk, and let us designate this measure of roundaboutness of production by b, such that $b=\frac{1}{1-c}$. On the other hand, since $\frac{a}{1-c}$ is the amount of labor required in the production of one unit of consumer good, the productivity of labor in the sense of quantity of consumer goods produced by a unit of labor should be the reciprocal of $\frac{a}{1-c}$, or $\frac{1-c}{a}$. Let us designate this by m, such that $m=\frac{1-c}{a}$. Then the equation (4) can be rewritten as follows:

$$\frac{F}{Q}=\frac{w}{m}.\ b\cdots\cdots\cdots\cdots(5)$$

The left-hand side of this equation is equivalent to the average capital/output ratio, and the right-hand side indicates that the capital/output ratio is directly proportional to the wage rate and the roundaboutness of production and is inversely proportional to the productivity of labor. This is the relationship which is quite similar to that revealed in the equation (1). Although there are certain differences in the concepts employed between the formulation which resulted in the equation (1) and that given here, the essential implication of a concept like capital/output ratio is revealed in both and can be expressed in the form of a function of the wage rate-dividedby-productivity, or w/m in (5).

6. The next problem to be dealt with is the relation between the investment/output ratio and the degree of capital intensity. As has been pointed out earlier, a widely accepted body of opinion as regards the investment programming of under-developed economies usually identifies the high investment/output ratio with the high degree of capital intensity, using both concepts in the sectoral sense as in the overall.

There is no objection to using the *average* capital/output ratio in the sectoral sense, so long as we are aware of the specific limitations of the concept in a particular application. Using the subscript i to indicate the sectoral use of various categories, we may write:

$$k_i=\frac{D_i+S_i}{Y_i}$$

where Y_i may be regarded as the sum of

wage payments in that sector (W_i) and the surplus value arising in that sector (Π_i). The average capital/output ratio in this formulation is, no doubt, quite similar to the concept of the degree of capital intensity, which may be designated by d and written as:

$$d_i=\frac{D_i}{W_i}$$

or the ratio of durable producers' structure and equipment employed in a certain sector and the wages-bill paid in that sector. If we write the relative share of workers in the new value created as η, such that $W=\eta Y$, we could easily relate the average capital/output ratio to the degree of capital intensity as follows:

$$k_i=\frac{D_i+S_i}{Y_i}=\frac{D_i}{Y_i}+\frac{S_i}{Y_i}=\eta d_i+\frac{S_i}{Y_i}$$

from which we may say that if inventories occupy a constant proportion of output and the relative share of workers remains unchanged, the average capital/output ratio will be roughly proportional to the degree of capital intensity.

The degree of capital intensity as formulated in this manner is essentially a sectoral concept and has its uses in economic analysis. But can we speak of a sectoral k', or a sectoral investment/output ratio? To the extent the economy is integrated in such a way that all the sectors are interrelated to each other, investment in any one sector will materialize itself in final output to which all the sectors contribute; and it will be difficult to speak of a sectoral k'. Take, for example, a new investment in cotton-spinning industry to the amount of I_c which can be associated with the increase in the value added in that industry to the amount of y_c. It is important to note in this case that y_c can be realized only if there goes on antecedent or concomitant expansion in, say, the spinning-machine industry *and* if it does not conflict with the concomitant investment in the competing industry, such as synthetic fibres. One could, if one wishes, refer to I_c/y_c as the investment/output ratio of the cotton spinning industry, but such a ratio could hardly be used as a criterion for investment decisions. It seems that if one wishes to use k' in the sense of reciprocal of productivity of investment, one has to confine the use of k' only in an overall sense except where a certain sector can be ascertained to be independent from other sectors.

Furthermore, it is quite *possible* that the overall investment/output ratio is lowered through the development of an industry with a high degree of capital intensity. This is *a priori* evident from the essential implication of the investment/output ratio developed earlier. Maurice Dobb has discussed this question extensively,[15] and we do not propose to go into the subject in this Note. It is only necessary to add that the concept of "capital" has been used in so many different meanings in the recent past and that it is about time that we make appropriate distinctions among, for example, capital in the sense of durable producers' structure and equipment, capital in the sense of a counterpart to saving (which could easily be an increase in inventories only as in the Marxian model of extended reproduction), capital in the sense of subsistence fund as in Böhm-Bawerk, and so on.

15 Maurice Dobb, "A Note on the so-called Degree of Capital-intensity of Investment in Under-developed Countries," *On Economic Theory and Socialism*, 1955, pp. 138—154.