

Marx's Theory of the Falling Tendency of the Rate of Profit¹⁾

—A Critique of Critiques—

Shigeto Tsuru

Though it has been recognized for long time that Marx's theory of the falling tendency of the rate of profit constitutes one of the most important elements in his theory of capitalistic crisis, very little progress has been achieved in the development of his theses along this line, either in that of the theory of profit-rate or in that of the theory of crisis. In fact, Marx himself did not leave any more than scattered fragments here and there so far as his theory of crisis is concerned. (The fourth section in chapter three of the second volume of his *Theorien über den Mehrwert* is perhaps the only exposition on crisis presented by Marx in any focalized fashion.) Besides, his main concern was to show the laws of the origin, development, and decline of the capitalistic mode of production, and he did not even tackle with such problems (which he intended to do so) as the theory of crisis in the stage of fully developed capitalism with credit system and large corporations.

Since Marx's death, we have witnessed, on the one hand, several critiques on his theory of the falling tendency of the rate of profit (notably by Tugan-Baranowsky and Bortkiewicz), and on the other hand, the develop-

ment in theories of crisis far outstripping the scope and depth which was of Marx's concern. For the former, we have not heard as yet the conclusive refutation of critiques by orthodox Marxists; and for the latter, Marxists have been left remotely behind, perhaps in accordance with their own intentions, in constructing positively their own theories on detail problems of crisis which have commanded the attention of modern business cycle economists,

The task which I have set upon myself within the scope of this paper is mainly to re-examine some of the critiques on Marx's theory of the falling tendency of the rate of profit; and I should like to postpone to another occasion the matter of assessing to what extent the theory still constitutes the basic element in the Marxian theory of crisis.

1. *Marx's theory of the falling tendency of the rate of profit:*

We may, for the sake of simplicity, formulate his theory in its bare mathematical form. Designating by:

- p' the average rate of profit,
- m' the rate of surplus value which we assume to be the same in all the industries,
- c the amount of constant capital (in value),
- v the amount of variable capital (in

1) This article was written in 1937 when the writer was a graduate student at Harvard University. It was not published before, and is here reproduced in its original form.

value),

R the ratio of c to v , or an expression of the organic composition of capital,

we get:

$$p' = m' \cdot \frac{v}{c+v}$$

or

$$p' = \frac{m'}{R+1}$$

Marx's theory states that in the process of capitalistic accumulation R has the tendency to increase, or that greater and greater quantity of machinery is introduced into all the industries with technical progress, and that increase in R will lower p' if m' remains the same. That is to say:

$$\frac{\partial p'}{\partial R} = \frac{1}{(R+1)^2} \cdot \left\{ (R+1) \frac{dm'}{dR} - m' \right\}$$

if $\frac{dm'}{dR} = 0$, then $\frac{\partial p'}{\partial R} < 0$.

Even if m' increases with R , p' will fall if

$$\frac{dm'}{dR} < \frac{m'}{R+1} = p'$$

or

$$\frac{R}{m'} \cdot \frac{dm'}{dR} < \frac{R}{R+1}$$

in other words, if "the elasticity of the rate of surplus value with respect to the organic composition of capital" is smaller than the number slightly less than one.²⁾ Further, even

2) We must be well aware that the positive content of such a mathematical formulation is limited by the extent to which we actually know the essential relations of concrete phenomena. A mathematical formulation may suggest further possible combinations of events. But whether our scientific pursuit (in social science at present) bears fruits by following the channels of investigation suggested through mathematical operations depends on our point of view concerning the task of science. Even if we settle this problem, the problem of finding in a complex actuality a counterpart to any mathematical condition or concept cannot be solved by mathematics until the actuality in

if m' increases beyond this limit as the result of a factor or factors which have caused the increase in R , it still remains true that p' has tended to fall, although the phenomenal outcome (in the sense of the outcome in the realm of phenomena) may be its rise. It was Marx's opinion that in the long run the rate of profit would fall.³⁾

2. Tugan-Baranowsky's criticism:

Tugan-Baranowsky based his criticism on the fact that the higher organic composition of capital, or the higher productivity of labor, necessarily means the shorter necessary labor-hours, thus the higher rate of surplus value.⁴⁾ On this ground, he contended that not only in the sphere of private enterprise as Marx himself admitted but also in the sphere of the entire industries in a society the organic composition of capital as such has nothing to do with the rate of profit and that the concept of surplus value has no efficacious significance.⁵⁾ His mathematical formulation of this criticism is as follows: Taking a as the total social product expressed in terms of use-value and b as the total social labor applied in producing a , and assuming that there is no constant capital and that the rate of surplus value is 100%, we have:

$$\frac{b}{2}v + \frac{b}{2}m = b, \quad \frac{a}{2}v + \frac{a}{2}m = a$$

If we introduce machinery and replace one

its totality could be condensed into a system of mathematical equations. But, alas! even the approximate formula describing the movement of the moon is said to cover one hundred and eighty pages.

3) *Das Kapital* (Adoratsky's edition), 3 Bd. p. 258.

4) cf. Tugan-Baranowsky, *Studien zur Theorie und Geschichte der Handelskrisen in England*. G. Fischer, Jena, 1901, p. 211.

5) *Ibid.* p. 216.

half of the labor force, it is likely that we shall increase the total social product and we must assume that the surplus product shall not decrease. Even taking the limiting case and assuming that the total product does not change, we have:

$$\frac{a}{4}c + \frac{a}{4}v + \frac{a}{2}m = a$$

But since $\frac{3}{4}a$ has been produced by $\frac{1}{2}b$ of the living labor, the value of the commodity is $\frac{2b}{3a}$, and value of the variable capital is equal to $\frac{b}{6}$. Thus we get:

$$\frac{b}{6}c + \frac{b}{6}v + \frac{b}{3}m = \frac{2}{3}b$$

In other words, while the rate of surplus value has increased from 100% to 200%, the rate of profit has remained the same at 100%. If we assume, as reality suggests, that a increases as the result of the introduction of machinery, the rate of profit is bound to rise.

Tugan-Baranowsky prides himself in having shown the basic mistake of Marx by the very use of the latter's logic, namely the labor theory of value, and suggests in turn that the problem of profit, to be treated correctly, should not be a problem in the theory of value, but that of aggregate surplus product in a society expressed in terms of use-values.⁶⁾

Tugan-Baranowsky's criticism has already been criticized by Bortkiewicz, Kautsky, and others. Bortkiewicz pointed out that Tugan-Baranowsky (1) obtained his result by changing the assumption as regards the rate of surplus value which Marx took to be constant and (2) took the most simplified case by avoiding completely those issues which arose out of the different compositions of capital

in different industries.⁷⁾

Even permitting Tugan-Baranowsky the license to ignore the above two points of Bortkiewicz, we do not recognize any blow dealt by Tugan-Baranowsky on Marx's theory. The basis of Tugan-Baranowsky's argument is that the rate of surplus value⁸⁾ is bound to rise as the result of improvement in technique. Marx admits this fully.⁸⁾ The problem, however, is not whether there is a logical possibility of rise in the rate of profit, but whether there is as an essential *Moment* in interacting relations of actuality a tendency for the rate of

7) Bortkiewicz, "Zur Berichtigung der grundlegenden theoretischen Konstruktion von Marx im dritten Band des 'Kapital'"; *Jahrbücher für Nationalökonomie u. Statistik*, 1907, vol. 89, pp. 334—335.

8) cf. the following passages from Marx: "The mass of profits contained in the individual commodities may nevertheless increase, if the rate of the absolute or relative surplus value grows. The commodity then contains less newly added labor, but its unpaid portion grows over its paid portion. However, this is the case only within certain limits. (*Capital* vol. 3. Kerr. ed. 1909, p. 265)

"The rate of profit might even rise, if a rise in the rate of surplus value were accompanied by a considerable reduction in the value of the elements of constant, and particularly of fixed, capital. But in reality, as we have seen, the rate of profit will fall in the long run. (*ibid.* p. 269)

In these passages, Marx admits the logical possibility of the rise in the rate of profit; but he hastens to add either that there are definite limits in concrete reality or that in the long run the rate of profit will fall.

When Tugan-Baranowsky says: "Er (Marx) hat eine ganze Reihe entgegenwirkender Ursachen angeführt, die die Wirkung des Gesetzes mehr oder weniger abschwächen sollten. Leider hat er unter diesen Ursachen auf die einzig wichtige nämlich die Steigerung der Produktivität der Arbeit, welche das ganze Gesetz aufhebt..... nicht hingewiesen," (p. 218) he is grossly misrepresenting Marx, because the first of the six counteracting causes which Marx mentions subsumes "die Steigerung der Produktivität der Arbeit."

6) *Ibid.* p. 208, pp. 218—226.

profit to fall.....a tendency which in turn invites all kinds of counteracting forces, registering on the surface of empirical statistics either fall or rise, as the case may be, of the rate of profit. Not the composite figure in the phenomenal realm, but the essential dynamic force in the social development is at issue.

So far as his arithmetical example is concerned, it is only by assuming, most unrealistically, that the rate of surplus value rises 100% while the total social capital even decreases that he is able to show the rise in the rate of profit.

3. Bortkiewicz's criticism :

The starting point of his criticism on Marx's theory of the falling tendency of the rate of profit is the rectification in Marx's *Umrechnung* from value-terms into price-terms. When Marx develops the concept of *Produktionspreis* as the sum of *Kostpreis* and the average profit (the individual quantity of capital multiplied by the average rate of profit), cost-price figures still in terms of value and not of production-prices. Marx himself admits, then, that "there is always the possibility of an error, if we assume that the cost-price of the commodities, of any particular sphere is equal to the value of the means of production consumed by it."⁹⁾ Bortkiewicz attempts to show that there is not only a mere possibility of an error but an actual one in ignoring the rectification. Because Marx does not make a necessary rectification, he can use the formula for the rate of profit :

$$p' = m' \cdot \frac{v}{c+v}$$

and later, using this formula, can state his

proposition that given the rate of surplus value the rate of profit is lower, the higher the organic composition of capital is. But the above formula for the rate of profit has to be modified once we introduce the necessary rectification in *Umrechnung*. Furthermore, proceeding from this rectification, it can be shown that if we assume three branches of production : I. intermediate goods, II. consumption goods to be consumed by workers alone, and III. consumption goods to be consumed by capitalists alone, it is only the organic compositions of the first two branches of production that the rate of profit is dependent on when the rate of surplus value is given. The consequence of this relation can be shown in a most telling way if we assume a case where the constant capital is absent in the second branch of production. In this case, the rate of profit turns out to be equal to the rate of surplus value, and this fact is in no way influenced by particular organic compositions of capital in the first and the third branches of production except within certain limits. In other words, for this case, the organic composition of the total social capital can grow up to a certain limit without changing the rate of profit at all if the rate of surplus value is assumed to be constant throughout.

Though Bortkiewicz seems to be of the opinion that Marx's mistake concerning the factors determining the height of the rate of profit can be traced back to the oversimplification in *Umrechnung*, thus to the adherence to the formula : $p' = m' \cdot \frac{v}{c+v}$, the two issues, in my opinion, do not have a necessary theoretical connection so far as Bortkiewicz's analysis is concerned. As has been stated before, the oversimplification in *Umrechnung* was recognized by Marx himself. He, however, tho-

9) *Capital*, vol. 3. p. 194.

ught that "our present analysis does not necessitate a closer examination of this point,"¹⁰⁾ because of two reasons: (1) Because, "this (the fact of deviations) always amounts in the end to saying that one commodity receives too little of the surplus-value while another receives too much, so that the deviations from the value shown by the prices of production mutually compensate one another. In short, under capitalistic production, the general law of value enforces itself merely as the prevailing tendency, in a very complicated and approximate manner, as a never ascertainable average of ceaseless fluctuations."¹¹⁾ (2) Because, he is mainly concerned with demonstrating that "the cost-price of a commodity is always smaller than its value."¹²⁾

The further rectification of *Umrechnung* which Bortkiewicz proposes constitutes, no doubt, a refinement. But for him, it is not only a refinement, but also a refutation of Marx on two scores: firstly, as regards the determination of the height of the rate of profit itself; and secondly, as regards Marx's

10) *Ibid.* pp. 194—195.

11) *Ibid.* p. 190.

Here is revealed one of the most important methodological maxims of Marx that "in the employment of the theoretical method (in political economy), the subject, society, must constantly be kept in mind as the premise from which we start." (*Critique of Political Economy*, Introduction, p. 295) Precision of physicists, he does not hope to attain. But instead of talking in terms of logical possibilities, he visualizes his subject matter, society, as concretely as he possibly can and eliminates those logical possibilities which in reality can only rarely be found. And also to visualize the general law as "the prevailing tendency" or as "a never ascertainable average of ceaseless fluctuations" is to emphasize the manner in which any law of social phenomena manifests itself.

12) *Ibid.* p. 195.

law of the falling tendency of the rate of profit.

Firstly, it is admittedly true that the rate of profit will turn out to be different when we make the rectification in *Umrechnung*. But when Bortkiewicz criticizes Marx in this connection for the discrepancy in the *Warenaustausch*, he is confusing the theoretical transference from the essential to the phenomenal with the actual transference in phenomena from year to year.

Secondly, it is yet to be seen whether the lack of the rectification has the necessary theoretical connection with the alleged mistake on the part of Marx concerning the falling tendency of the rate of profit. Bortkiewicz's criticism amounts to saying, though he did not formulate it in this way, that the average rate of profit is a function only of the organic composition of capital of those branches of production which either directly or indirectly produce the consumption goods consumed by workers alone.¹³⁾ In the first place, this proposition does not arise out of the rectification, but out of the recognition that the organic compositions of those branches of production which are exclusively used in the production of luxury goods (to be consumed only by capitalists) do not affect the average rate of profit.¹⁴⁾ Therefore, in this regard, the matter of rectification in *Umrechnung* should be theoretically separated from the question of the rate of profit. In the second place, the proposition does not consti-

13) The case of constant capital in the branch II. being zero (a very unrealistic case!) has been shown by Bortkiewicz to give rise to the profit equal to the rate of surplus value. This is obvious once we formulate his thesis in the above fashion.

14) Marx was aware of this point. See *Capital*, vol. I, p. 346.

tute a refutation of Marx's theory of the falling tendency of the rate of profit. Concretely, the case which Bortkiewicz visualizes as upsetting Marx's theory is as follows:

- (1) the organic composition of the branch II. falls,
- (2) the organic composition of the branch III. rises,
- (3) the latter rises more than the former, so that the organic composition of the total social capital rises,
- (4) the rate of surplus value remains the same,
- (5) but the rate of profit rises.

Such a sequence of events is certainly *logically possible*. But will it ever happen in actuality? The organic composition of the branch II. has the constant tendency to rise, while that of the III. either stays stationary or falls; and even when the contrary happens, it is most unlikely that the rise in the III. will more than offset the fall in the II.

Mathematically, using Bortkiewicz's notation¹⁵⁾, we may formulate the factors determining the rate of profit as follows:

$$\sigma = \frac{-(v_2C + Ve_1) + \sqrt{(v_2C - Ve_1)^2 + 4VCe_2v_1}}{2(c_2v_1 - v_2c_1)}$$

This formula can be used even when c_2 is

zero, contrary to the Bortkiewicz's. If the organic compositions of the I. and II. are the same, in other words, if $c_2v_1 = v_2c_1$, the formula simplifies itself to:

$$\sigma = \frac{VC}{v_2C + Ve_1}$$

It should be emphasized here that "the organic composition" is used in Marx's sense, namely in terms of *value-compositions* and not of *price-compositions*.

If we depart the realm of value-terms, we have the following relation between the rate of profit and the *price* organic composition of the total social capital:

$$\rho = \frac{zM}{xC + yV} = \frac{\frac{zM}{yV}}{R + 1} \quad \left(R = \frac{xC}{yV} \right)$$

If $\frac{M}{V} = m' = \text{constant}$,

$$\rho = \frac{zm'}{y(R + 1)}$$

then,

$$\frac{\partial \rho}{\partial R} = \frac{m'}{(R + 1)^2 y^2} \cdot \left\{ y(R + 1) \frac{dz}{dR} - z \left(y - [R + 1] \frac{dy}{dR} \right) \right\}$$

In order that $\frac{\partial \rho}{\partial R} < 0$

$$(R + 1) \left(y \frac{dz}{dR} - z \frac{dy}{dR} \right) - zy < 0$$

or

$$\frac{R}{z} \cdot \frac{dz}{dR} - \frac{R}{y} \cdot \frac{dy}{dR} < \frac{R}{R + 1}$$

It is quite likely, as can be shown from concrete examples, that such condition is satisfied in actuality.

4. Professor Shibata's criticism:

Kautsky had tried to refute Tugan-Baranovsky's criticism by pointing out that Tugan-Baranovsky's criticism applied *only* in the case where the rise in the productivity of labor does *not* affect the price of products

15)

c_1 the constant capital in value of the branch I.
 c_2 " " " II.
 c_3 " " " III.
 v_1 the variable capital in value of the branch I.
 v_2 " " " II.
 v_3 " " " III.
 z the ratio of price to value of the products of branch I.
 y " " " II.
 z " " " III.
 ρ the average rate of profit
 M = the total surplus value
 $V = v_1 + v_2 + v_3$
 $C = c_1 + c_2 + c_3$
 $\sigma = 1 + \rho$

and that if we take such effect into account the rate of profit is bound to fall.¹⁶⁾ Professor Shibata's contention is that the very case of falling prices involves the rising rate of profit.¹⁷⁾

Professor Shibata, like Bortkiewicz, deals first with the question of *Umrechnung*. He also proposes a rectification, but arrives at the conclusion that if we carry the *logical* process of trans-calculation from value-terms to price-terms step by step, we obtain as a final price-equilibrium of this process the result which is identical with the case when we start from the very beginning in terms of prices. He, however, does not dismiss the consideration of value-terms entirely. It is his opinion that the question of value is germane to the matter of a priori normative attitude and that in so far as the determination of economic phenomena is concerned it is irrelevant whether we take value-terms into consideration or not.¹⁸⁾ Thus he proceeds to prove his original proposition by reasoning from the outset in terms of prices alone.

His analysis in this connection can be divided into three steps:

(1) The average rate of profit is a function of the following variables alone: a. production-coefficients of the commodity constituting real-wages, b. production-coefficients

16) Kautsky, "Krisentheorien" *Neue Zeit*, xx. Jahrg. 2. Bd. p. 43.

17) Or more accurately, in his words, that "changes in the coefficients of production, in so far as such changes bring about reduction in the expenses of production, give rise necessarily to the rate of profit, even when accompanied by the rise in the organic composition of capital." (Shibata, *Theoretical Economics*, vol. I. or *Rironkeizai-gaku* p. 228))

18) *Ibid.* p. 203. also cf. Shibata, "The Meaning of the Theory of Value in Theoretical Economics" *Kyoto Univ. Economic Rev.* Dec. 1933. p. 68.

of the means of production used in producing the above commodity, c. production-coefficients of the means of production used in producing both of the above commodities, (and so on), d. real-wage of the labor power relevant for the above, and e. the period of turn-over of capitals used in producing the above commodities.¹⁹⁾

(2) Changes in the production-coefficients, in so far as such changes bring about reduction in the expenses of production, give rise necessarily to a higher rate of profit than before, even when accompanied by the rise in the organic composition of capital.²⁰⁾

(3) If the higher price composition of capital and the lower average rate of profit occur simultaneously, such a coincidence is likely to be due to the following combination of events; namely, the increase in real-wage (or the shortening of the working day), which leads to the lower average rate of profit and the lower price composition of capital, and the improvement in the methods of production, which leads to the higher average rate of profit and the higher price composition of capital, take place simultaneously, the second effect of the latter being stronger than the second effect of the former (thus the composite effect on the social capital is the *higher* price composition) and the first effect of the former being stronger than the first effect of the latter (thus the composite effect is the *lower* average rate of profit.)²¹⁾

Professor Shibata proves the above series of propositions with the aid of arithmetics under certain simplifying assumptions. We shall attempt to reformulate his proof in a

19) Cf. *ibid.* p. 226.

20) Cf. *ibid.* p. 228.

21) Cf. *ibid.* pp. 244—245.

more general way.²²⁾

Assume, for the sake of simplification, that there are only five kinds of products in society: (1) money, (2) consumers' goods to be consumed only by workers, (3) consumers' goods to be consumed only by capitalists, (4) producers' goods to be used in the production of itself and (2), and (5) producers' goods to be used in the production of itself, (1), and (3). Assume also that the real-wage is invariant and that the period of turn-over is uniformly one year in all the branches of production. Then

- (1) $(c_0k_2 + a_0wp_1)(1+i) = 1$
- (2) $(c_1k_1 + a_1wp_1)(1+i) = p_1$
- (3) $(c_2k_2 + a_2wp_1)(1+i) = p_2$
- (4) $(\gamma_1k_1 + \alpha_1wp_1)(1+i) = k_1$
- (5) $(\gamma_2k_2 + \alpha_2wp_1)(1+i) = k_2$

Because (2) and (4) reduce themselves to two simultaneous equations with two unknowns (i and k_1/p_1), we can determine i in terms of known variables, $c_1, a_1, \gamma_1, \alpha_1, w$, thus proving the first point,

$$i = \frac{-2w(a_1\gamma_1 - c_1\alpha_1) + wa_1 + \gamma_1}{2w(a_1\gamma_1 - c_1\alpha_1)} - \sqrt{\frac{(wa_1 + \gamma_1)^2 - 4w(a_1\gamma_1 - c_1\alpha_1)}{4w^2(a_1\gamma_1 - c_1\alpha_1)^2}} \dots \dots (6)$$

if $\frac{c_1}{a_1} = \frac{\gamma_1}{\alpha_1}$

$$i = \frac{1}{(wa_1 + \gamma_1)} - 1 \dots \dots \dots (7)$$

Next, taking the simplest case where the price compositions of capital in (2), (3), (4), and (5) are the same, we can show that the rise in the price composition accompanied by the fall in all the prices is bound to raise the rate of profit. (In this case, we must assume that the money-producing branch of production remains unaffected, in order that we can isolate the effect of the rise in the price composition of capital.)

If we assume

$$c_1 = c_2 = \gamma_1 = \gamma_2 = c$$

$$a_1 = a_2 = \alpha_1 = \alpha_2 = a$$

(7) simplifies further to

$$i = \frac{1}{wa + c} - 1$$

or taking $R^{23)} = c/a$

$$i = \frac{1}{a(w + R)} - 1 \dots \dots \dots (8)$$

Under the above assumptions, it can be easily shown that

$$p_1 = p_2 = k_1 = k_2$$

Therefore from (1)

$$p_1 = \frac{1}{(c_0 + a_0w)(1+i)}$$

substituting (8)

$$p_1 = \frac{a(w + R)}{(c_0 + a_0w)} \dots \dots \dots (9)$$

Differentiating (9) and (8) with respect to R

$$\frac{\partial p_1}{\partial R} = \frac{1}{c_0 + a_0w} \left\{ a + (w + R) \frac{da}{dR} \right\}$$

23) When we assume that w is invariant and p 's and k 's vary in constant proportions, R_1 as defined here, can be taken as an expression of the organic composition of capital.

22) We shall follow, with a few exceptions, Professor Shibata's notations:

- i average rate of profit
- p_1 the price of the first kind of consumers' goods, or (2)
- p_2 " " second " " or (3)
- k_1 " " first kind of producers' good or (4)
- k_2 " " second " " or (5)
- w the real-wage
- c_0 the production-coefficient of (5) in producing money
- c_1 " " (4) " (2)
- c_2 " " (5) " (3)
- γ_1 " " (4) " (4)
- γ_2 " " (5) " (5)
- a_0 " " of labor power " money
- a_1 " " " " (2)
- a_2 " " " " (3)
- α_1 " " " " (4)
- α_2 " " " " (5)

$$\frac{\partial i}{\partial R} = - \frac{1}{a^2(w+R)^2} \left\{ a + (w+R) \frac{da}{dR} \right\}$$

If $\frac{\partial p_1}{\partial R} < 0$, then

$$a + (w+R) \frac{da}{dR} < 0$$

and if $a + (w+R) \frac{da}{dR} < 0$, then

$$\frac{\partial i}{\partial R} > 0$$

Thus the second point is proved. The third point is a corollary the proof of which we shall dispense with here.

In Professor Shibata's critique, the problem of *Umrechnung* and that of the rate of profit are theoretically not independent of each other, because it is only on the basis of his solution of the former that the latter can be discussed in terms of prices alone. Thus our critique of his critique also starts from the problem of *Umrechnung*.

It is not sufficiently realized by Professor Shibata that to regard the problem of value exclusively as that of normative attitude and to consider price-relations as sufficient in determining economic phenomena is to minimize the controlling efficacy which Marx attributed to value-relations in determining long-run phenomena in capitalist society and to turn our attention towards more precise logical analysis of short-run price phenomena. He seems to put too much emphasis on the causal efficacy of capitalist's mentality which is tuned for maximizing the price rate of profit and not the rate of surplus value. The fact that on the concrete phase of social phenomena this is the case does not preclude the long-run controlling efficacy of underlying value relations which manifest themselves in the realm of *phenomena* only in an approximate manner or "as a never ascertainable average of ceaseless fluctuations." The fact

that Professor Shibata gets the same result whether he starts out with value relations or with price relations is to be explained *not as a theoretical necessity* but thanks to a kind of *mathematical operation* which he uses as a counterpart to his theoretical requirement of pursuing the value-principle to the utmost.

Granting, however, that Professor Shibata's solution of the problem of *Umrechnung* is valid, we can indicate a certain discrepancy which arises in his theoretical structure. The simplified case which he assumes in proving his proposition concerning the tendency of the rate of profit is precisely the case which does not require the *Umrechnung* at all, because organic compositions of capital in all the branches of production are identical. Thus in this case, price is identical with value. By assumption, therefore, we may discuss this case in terms of value and not of price. For this purpose, we shall introduce a value equation incorporating Marx's labor theory of value and use it in place of money equation or our previous equation (1). Using the same symbols as before, though *p*'s and *k*'s now in terms of value and not of price, we have:

$$(10) \quad wp_1 + i(Rk_1 + wp_1) = 1$$

$$(11) \quad (ck_1 + awp_1)(1+i) = p_1$$

$$(12) \quad (ck_2 + awp_1)(1+i) = k_1$$

From (11) and (12),

$$i = \frac{1}{wa+c} - 1 = \frac{1}{a(w+R)} - 1 \dots \dots \dots (13)$$

From (10) $(p_1 = k_1)$

$$p_1 = \frac{1}{w+i(R+w)} = \frac{1}{w+(R+w) \left\{ \frac{1}{a(w+R)} - 1 \right\}} = \frac{a}{1-aR} \dots \dots \dots (14)$$

Differentiating (14) and (13) with respect to *R*,

$$\frac{\partial p_1}{\partial R} = \frac{\frac{da}{dR} + a^2}{(1 - aR)^2}$$

$$\frac{\partial i}{\partial R} = -\frac{1}{a^2(w+R)^2} \left\{ a + (w+R) \frac{da}{dR} \right\}$$

In order that $\frac{\partial p_1}{\partial R} < 0$

$$\frac{da}{dR} + a^2 < 0 \quad \text{or} \quad -\frac{da}{dR} > a^2 \quad \dots\dots(15)$$

In order that $\frac{\partial i}{\partial R} < 0$

$$a + (w+R) \frac{da}{dR} > 0 \quad \text{or} \quad \frac{a^2}{wa+c} > -\frac{da}{dR} \quad \dots(16)$$

Since $\frac{da}{dR} < 0$ and $0 < wa+c < 1$, there is a wide range of possibility where (15) and (16) are compatible. And yet this combination of events is clearly not the one visualized by Professor Shibata in his third point, because we have assumed the real wage to be invariant.

The discrepancy we have revealed in Professor Shibata's theoretical structure leads us

to question how it arose. The difference between the two cases lies in using the money equation in one case and the value equation in the other case. From the nature of the money equation which Professor Shibata uses, it is obvious, without any further mathematical operation or theoretical perusal, that *i* has to increase when *p* and *k* decrease. No matter how *i* may be determined, the above result must follow, so long as we keep *w*, *e*₀, and *a*₀ invariant. Prof. Shibata would defend the use of this equation on the ground that we must isolate the effect of the rise in the organic composition of capital by keeping the base of calculation unchanged. To keep this unchanged, however, is one thing, while to let it provide with the *effective* standard of measurement is another thing. Thus we are led to cast grave doubt on the theoretical necessity which Professor Shibata claims for his proposition.